

Geomechanics

LECTURE 9

HYDRO-MECHANICAL MODELLING AND RECOMMENDATIONS FOR MODELLING IN GEOTECHNICS

CFMS

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Laboratory of soil mechanics - Fall 2025

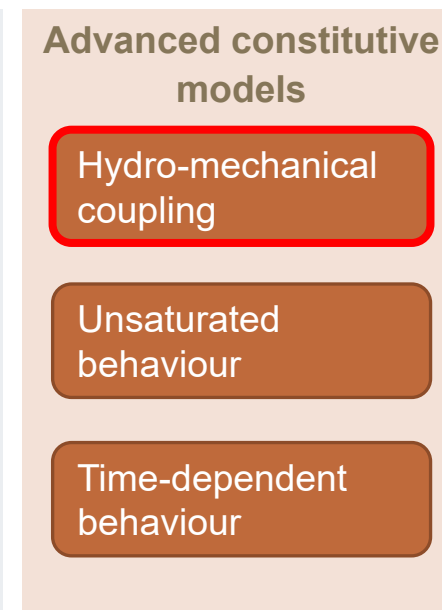
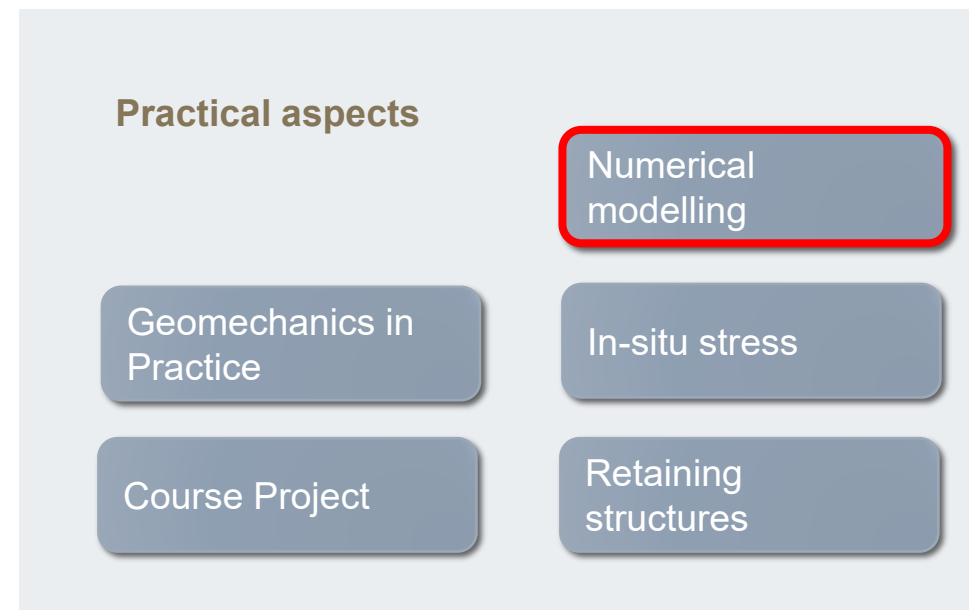
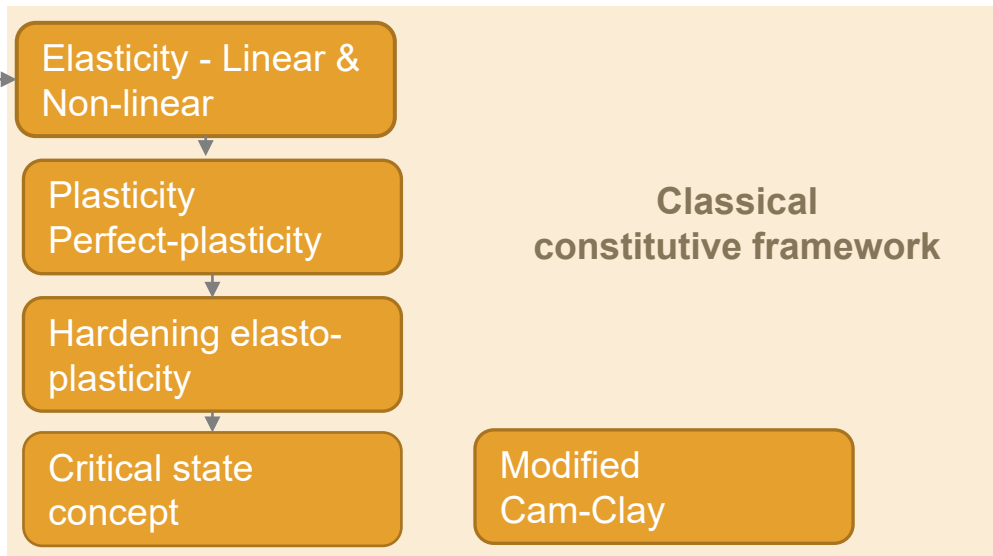
17.11.2025

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<https://etc.ch/4gMg>

Basic concepts



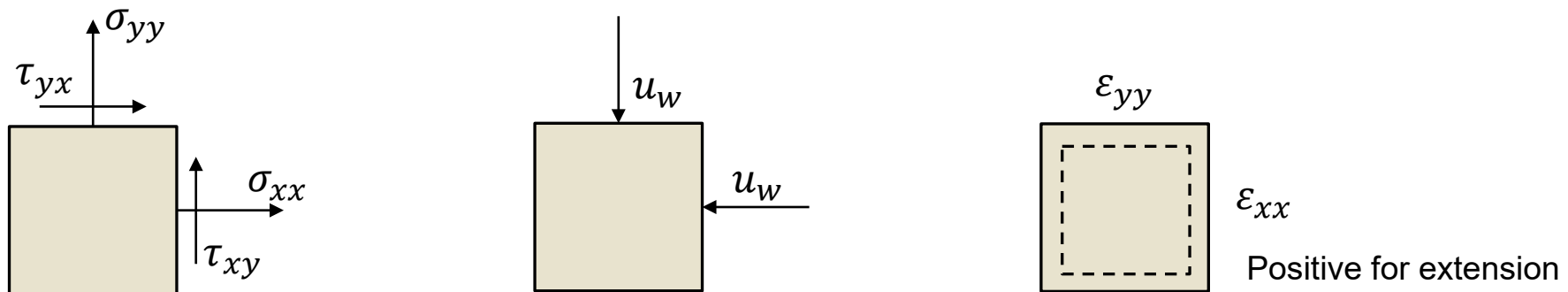
Topics

From constitutive modelling to Boundary Value Problems

- Constitutive models apply at REV level.
- For solving geotechnical systems (e.g. design of foundations and retaining structures, analysis of settlements induced by building, tunnel excavations, ...), the constitutive models must be integrated in a more general formulation to account for stresses and strains evolutions in the entire area of interest.

Hydro-mechanical formulation (u-p formulation)

- Set of equations which account at the same time for the mechanical response of geomaterials and the role of water (in terms of pressure and flow).
- Several formulations exist, accounting for different levels of complexity. In the following, we'll see a common form (the so-called u - p formulation) intended for soils in dry and/or saturated states.
- Sign conventions to be checked for each formulation / numerical code; for the following set of equations:



Variables (n. of unknowns)

σ_{ij} : total stresses (6)

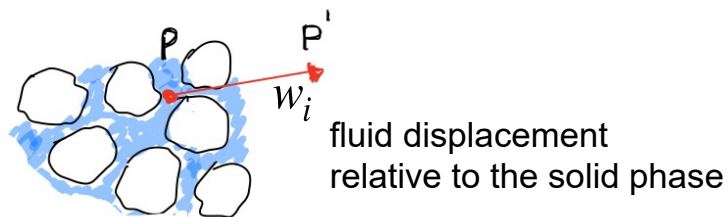
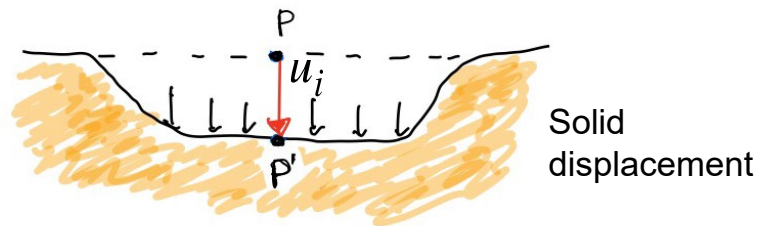
u_w : pore water pressure (1)

σ'_{ij} : effective stresses (6)

ϵ_{ij} : deformations (6)

u_i : solid skeleton displacements (3)

w_i : fluid displacement (3)



25 unknowns vs 25 equations

Equations (n. of equations)

Momentum (3)

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0 \quad (\text{no acceleration terms}), f_i \text{ are body forces}$$

Effective stress definition (6)

$$\sigma'_{ij} = \sigma_{ij} + u_w \delta_{ij} \quad (\text{Terzaghi's definition, saturated soil})$$

Constitutive relationship (6)

$$\dot{\sigma}'_{ij} = D_{ijhk} \dot{\epsilon}_{hk}$$

Compatibility of displacements (6)

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

Darcy's law (3)

$$\dot{w}_i = -k_{ij} \frac{\partial}{\partial x_j} \left(\frac{u_w}{\gamma_w} + z_e \right)$$

Continuity of fluid phase (1)

$$\frac{\partial \dot{w}_i}{\partial x_i} = -\dot{\epsilon}_{vol} \quad (\text{compressibility of fluid and solid neglected})$$

Example 1 : Dry soil

σ_{ij} : total (= effective) stresses (6)

ϵ_{ij} : deformations (6)

u_i : solid skeleton displacements (3)

Momentum (3)

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0 \quad (\text{no acceleration terms}), f_i \text{ are body forces}$$

Constitutive relationship (6)

$$\delta \sigma'_{ij} = D_{ijhk} \delta \epsilon_{hk}$$

Compatibility of displacements (6)

$$\epsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

15 unknowns vs 15 equations

Example 2 : Saturated soil subjected to change in total stresses

The new total stress field must be equilibrated:

$$\frac{\partial \sigma_{ij}}{\partial x_j} + f_i = 0$$

Changes in total stresses imply (general case) changes in effective stresses and pore water pressure:

$$\sigma'_{ij} = \sigma_{ij} + u_w \delta_{ij}$$

Deformations are linked to changes in effective stresses

$$\dot{\sigma}'_{ij} = D_{ijhk} \dot{\epsilon}_{hk}$$

Changes in pore water pressure imply change in fluid velocity:

$$\dot{w}_i = -k_{ij} \frac{\partial}{\partial x_j} \left(\frac{u_w}{\gamma_w} + z_e \right)$$

Continuity eq. links change in fluid velocities to deformations:

$$\frac{\partial \dot{w}_i}{\partial x_i} = -\dot{\epsilon}_{vol}$$

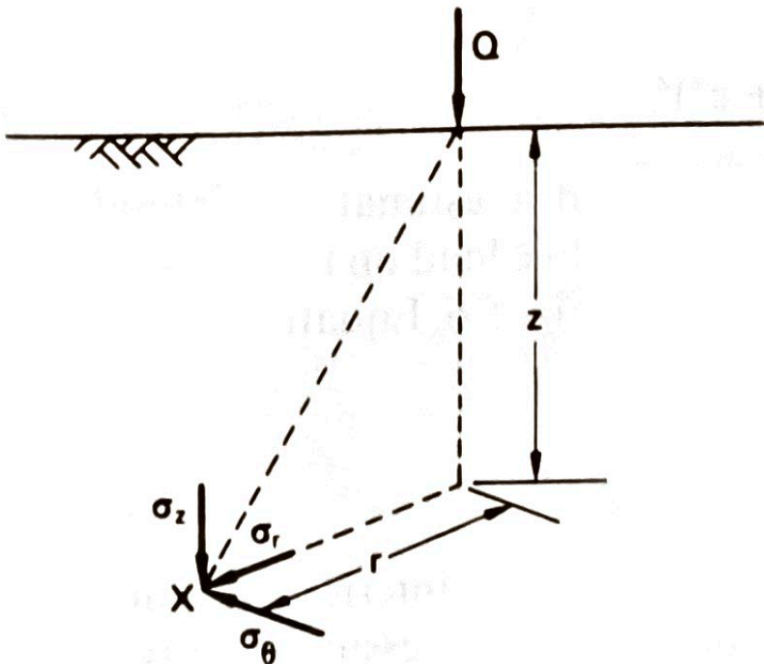
Need to solve all the equations simultaneously: **Hydro-Mechanical coupled problem**
(e.g. Consolidation)

From constitutive modelling to Boundary Value Problems

- Constitutive models apply at REV level.
- For solving geotechnical systems (e.g. design of foundations and retaining structures, analysis of settlements induced by building, tunnel excavations, ...) the constitutive models must be integrated in a more general formulation to account for stresses and strains evolutions in the entire area of interest.
- The resulting set of equations can be solved
 - Analytically, only in very simple cases;
 - Numerically, typically by Finite Element (FE) modelling or Finite Difference (FD) modelling.
- Commercial software for the engineering practice exist with both numerical approaches:
 - FE approach: e.g. Plaxis, ZSoil, RS2, ...
 - FD approach: Flac2D, ...
- Academic codes also exist which offer more advanced possibilities.

Analytical solutions

- Available for simple (i) geometries, (ii) constitutive laws (often elasticity), (iii) loading conditions.
- Very few solutions exist accounting for hydro-mechanical couplings (e.g. 1D consolidation theory).
- An example of the stress induced by a point load on the surface on an elastic semi-infinite space:



from Craig (1983)

$$\sigma_z = \frac{3Q}{2\pi z^2} \left\{ \frac{1}{1 + (r/z)^2} \right\}^{5/2}$$

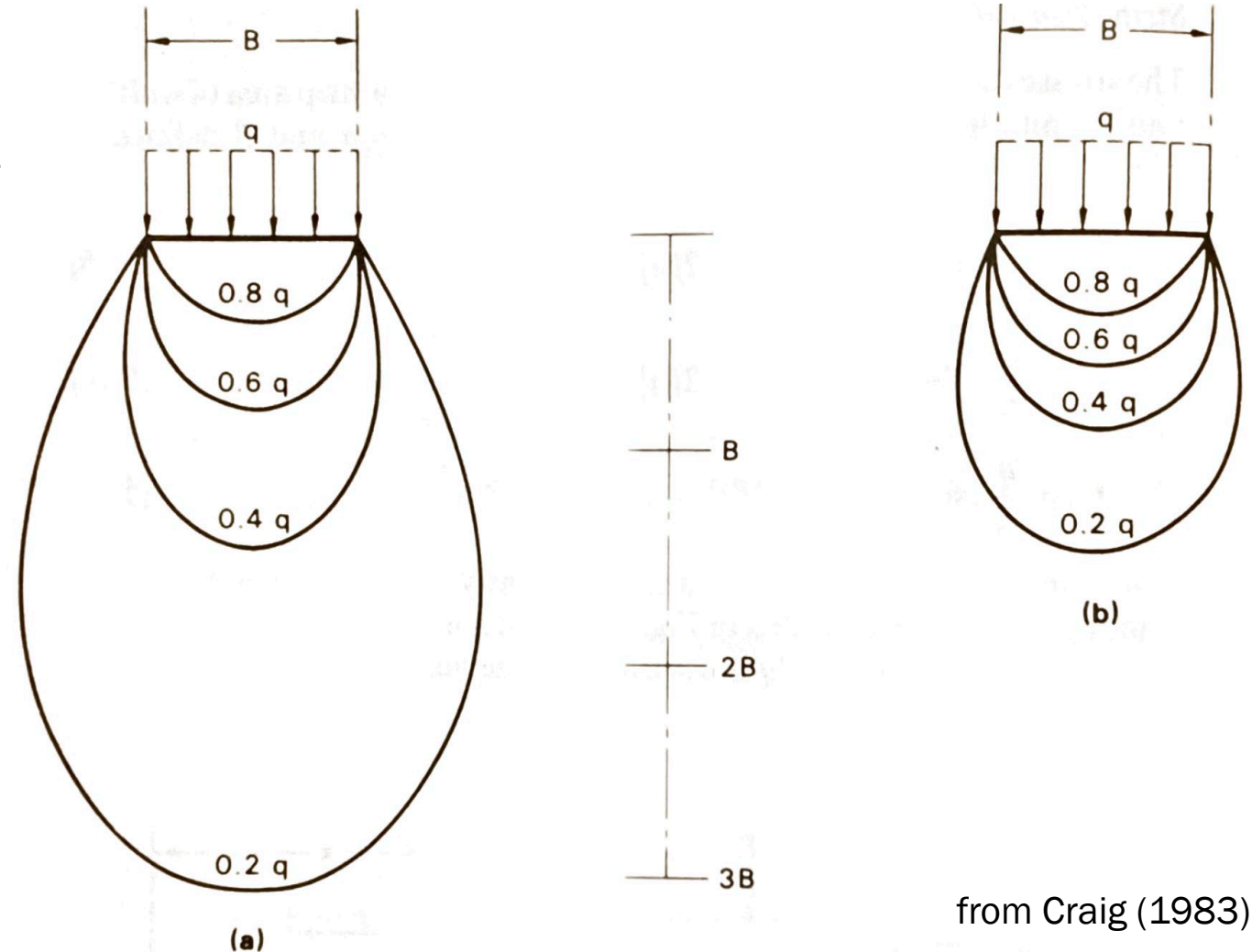
$$\sigma_r = \frac{Q}{2\pi} \left\{ \frac{3r^2 z}{(r^2 + z^2)^{5/2}} - \frac{1 - 2\nu}{r^2 + z^2 + z(r^2 + z^2)^{1/2}} \right\}$$

$$\sigma_\theta = -\frac{Q}{2\pi} (1 - 2\nu) \left\{ \frac{z}{(r^2 + z^2)^{3/2}} - \frac{1}{r^2 + z^2 + z(r^2 + z^2)^{1/2}} \right\}$$

$$\tau_{rz} = \frac{3Q}{2\pi} \left\{ \frac{rz^2}{(r^2 + z^2)^{5/2}} \right\}$$

Analytical solutions

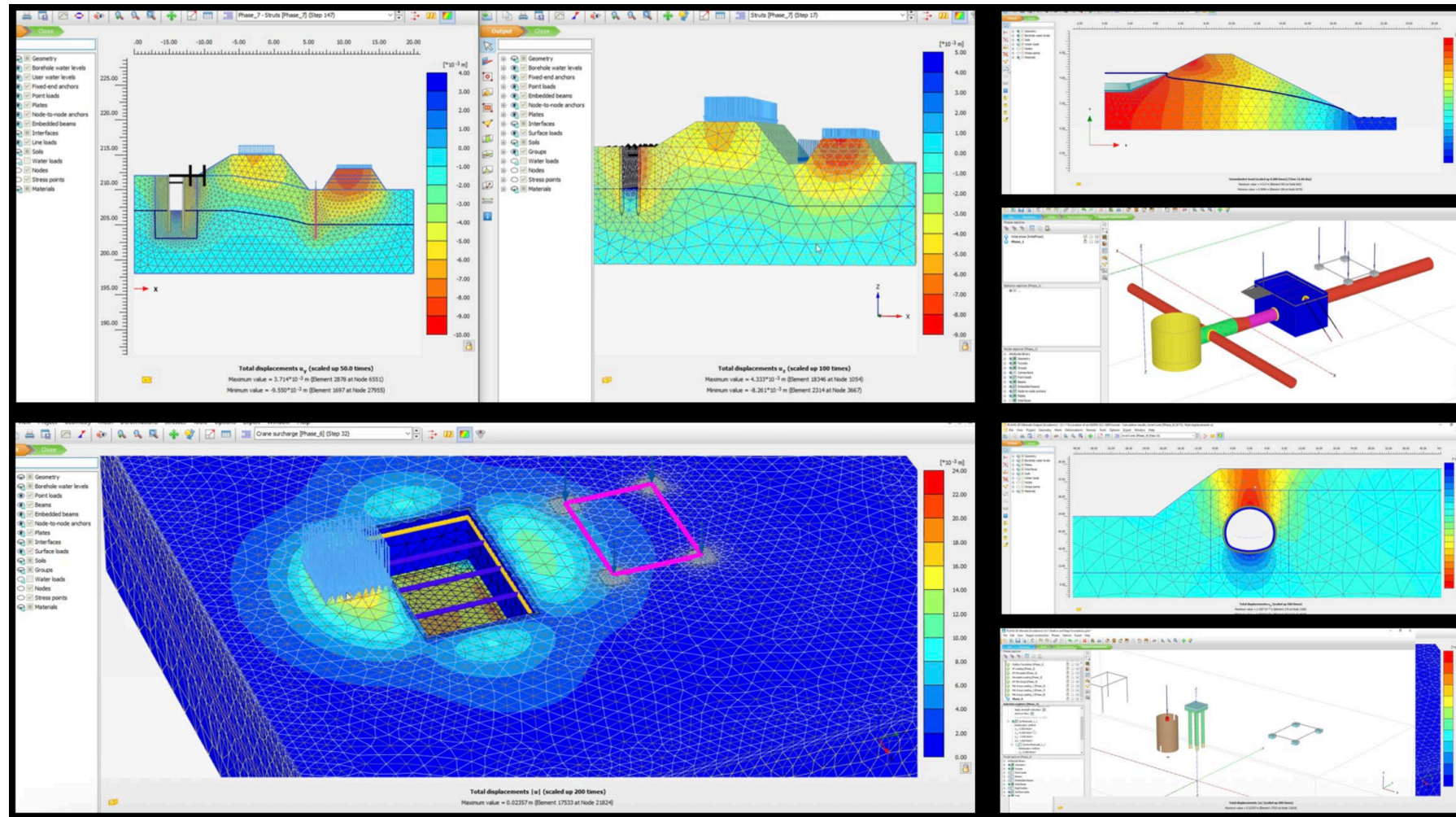
- They are useful to get a first estimation of the volumes affected by the changes in the boundary conditions.
- They can be used to validate numerical formulations



from Craig (1983)

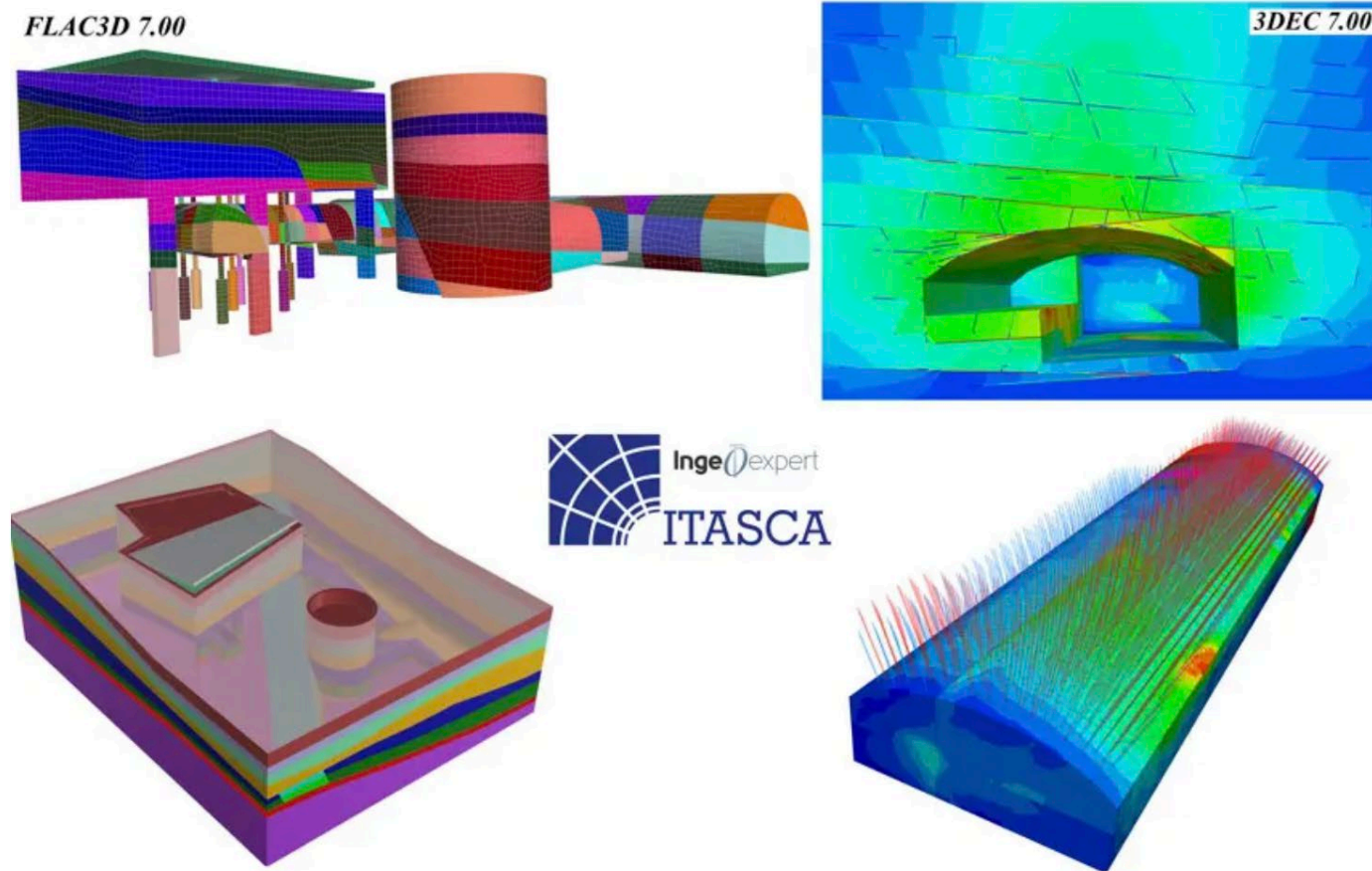
Fig. 5.8 Contours of equal vertical stress: (a) under strip area, (b) under square area.

Numerical analyses



Examples of analyses performed with the FE software Plaxis.

Numerical analyses



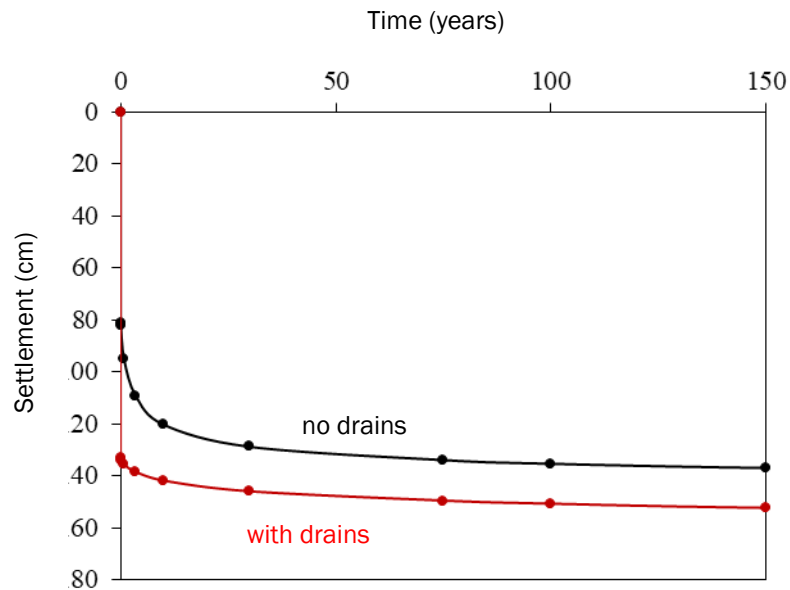
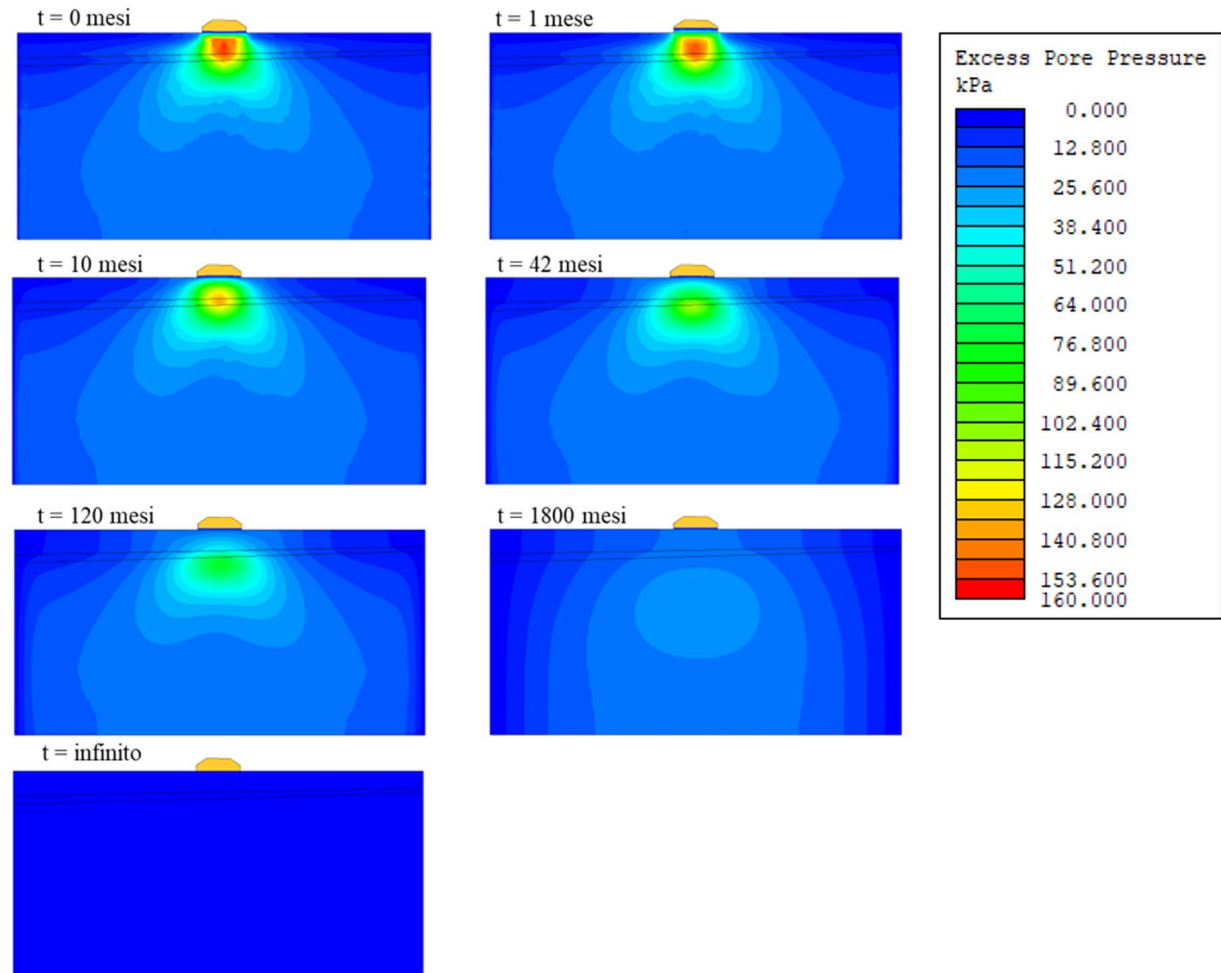
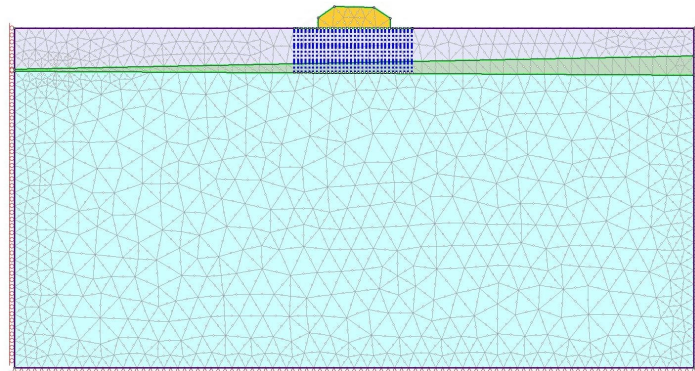
Examples of analyses performed with the FD software Flac.

Strategies of numerical modelling

- Steady state vs transient analyses
 - Steady state: all variations with respect to time are null. All variables are directly in equilibrium with the boundary conditions. Usually employed for the initial state and the final (long-term) condition.
 - Note that hydrostatic condition is a particular case of steady state.

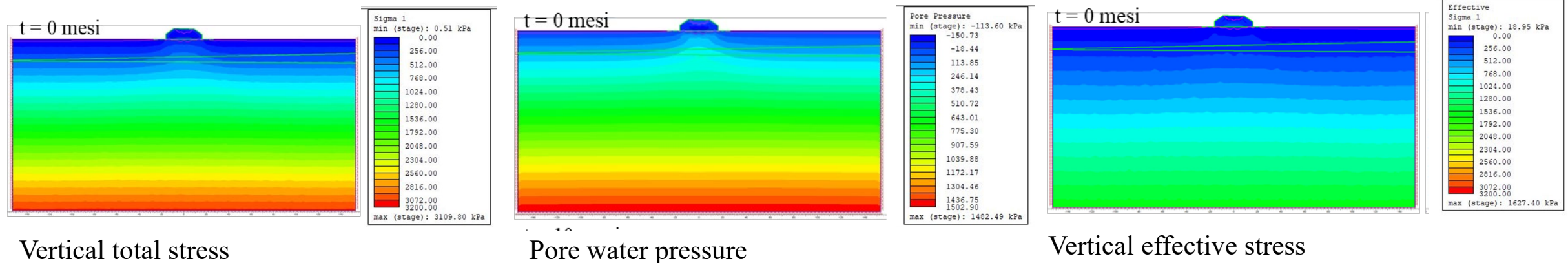
Strategies of numerical modelling

- Transient : Evolutions with time are explicitly considered (e.g consolidation analyses)



Strategies of numerical modelling

- Undrained analyses can be performed in two alternative ways:
 - Using the complete u-p formulation (short time to be considered): changes in pore water pressures are computed so that changes in effective stresses are evaluated. Constitutive models and parameters in terms of effective stresses are used.



- The geomaterials are modelled as single-phase materials obeying to total stresses. Constitutive law and parameters are defined in terms of total stresses (e.g. undrained Poisson's ratio for linear elasticity is $\nu_u = 0.5$).

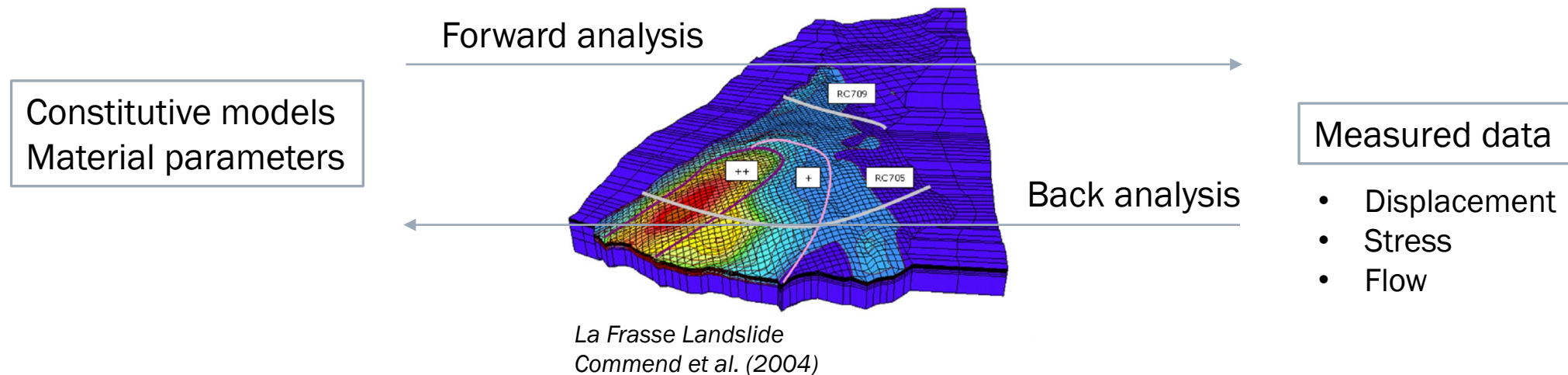
Goals of numerical models

Forward analysis

- Verify geotechnical work design with respect to limit states in design codes
- Test different design variants before construction phase starts

Back analysis

- Provide a theoretical explanation for observed behavior
- Improve the numerical model (constitutive laws, parameters)

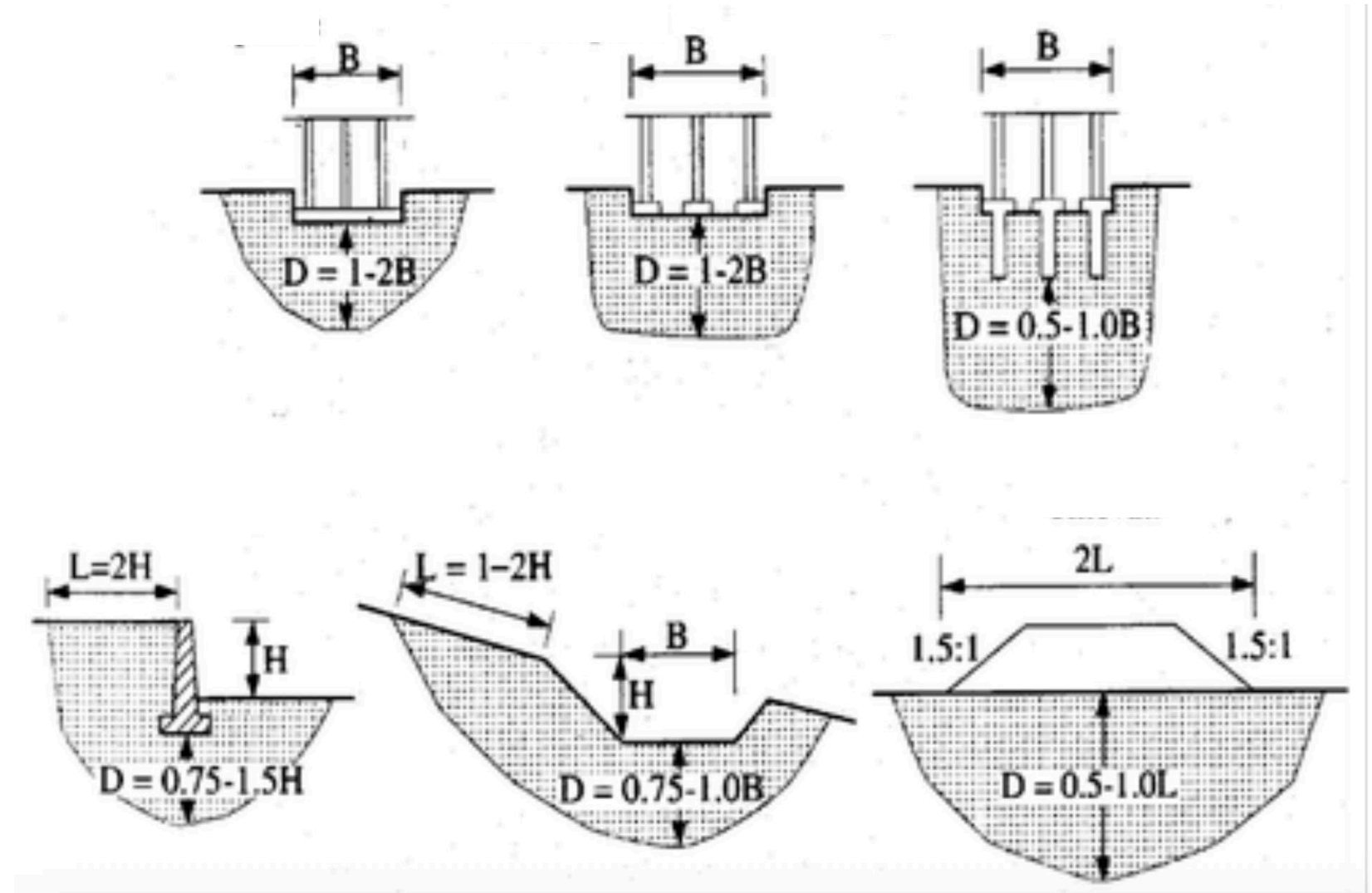


Key elements in the construction of a numerical model

- Selection of model elements and geometry (types of soil, structural elements, 2D-3D, ...)
- Choice of suitable constitutive models
- Modelling of structural interactions
- Definition of the initial state and boundary conditions
- Definition of the project phases
- Consideration of the effect of environmental factors and how to couple them with the mechanical behavior

Model geometry

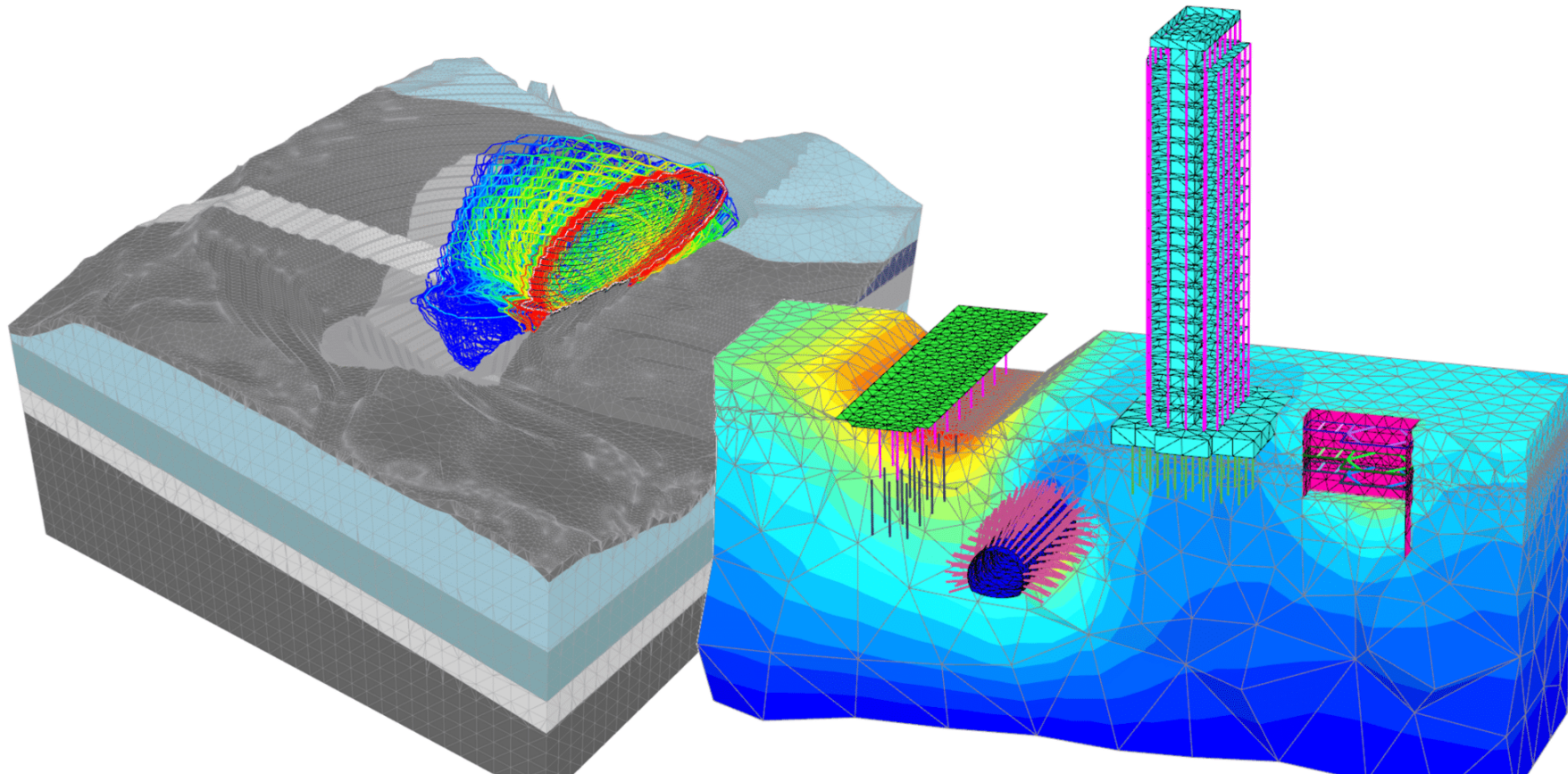
- The model geometry should account for all phenomena that affect the behavior of the studied problem, including the surrounding structures



Model geometry

3D analyses are more and more common, but they can often lead to more complex models than necessary

- If the geometry is complex and requires 3D model, other aspects of the analysis can be simplified



Model geometry

2D simplifications come from the consideration of symmetric properties or invariance of the problem

- Most of geotechnical problems are assumed in plane strain or axisymmetric conditions

- **Plane strain** conditions

Often, 2D analyses are performed on multiple sections to avoid 3D model

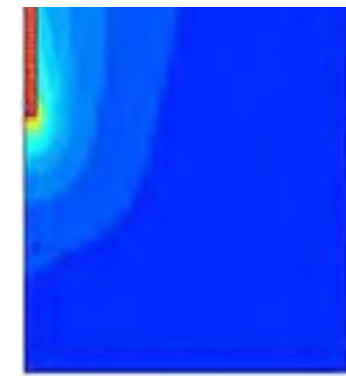
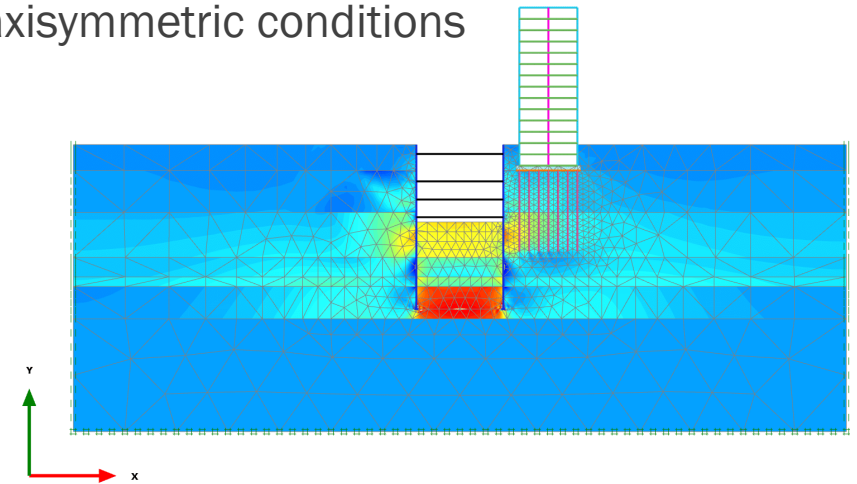
- Strip foundations
- Deep tunnels
- Retaining structures (walls, diaphragms)

- **Axi-symmetric** conditions

! Horizontal axis of symmetry neglects gravity !

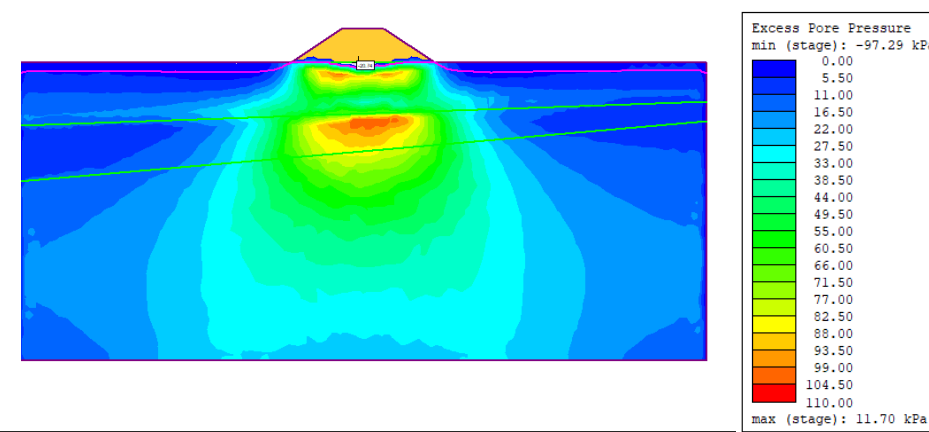
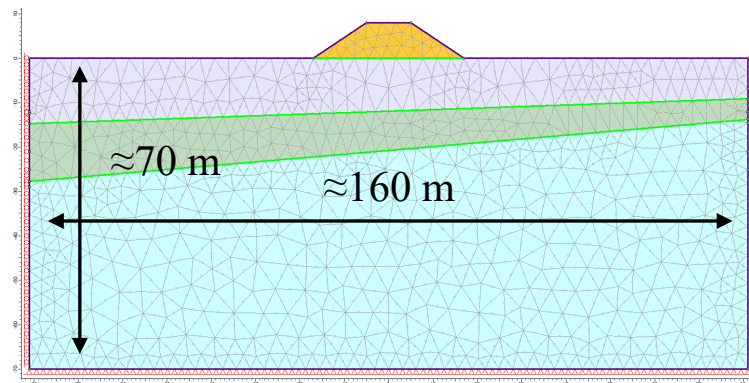
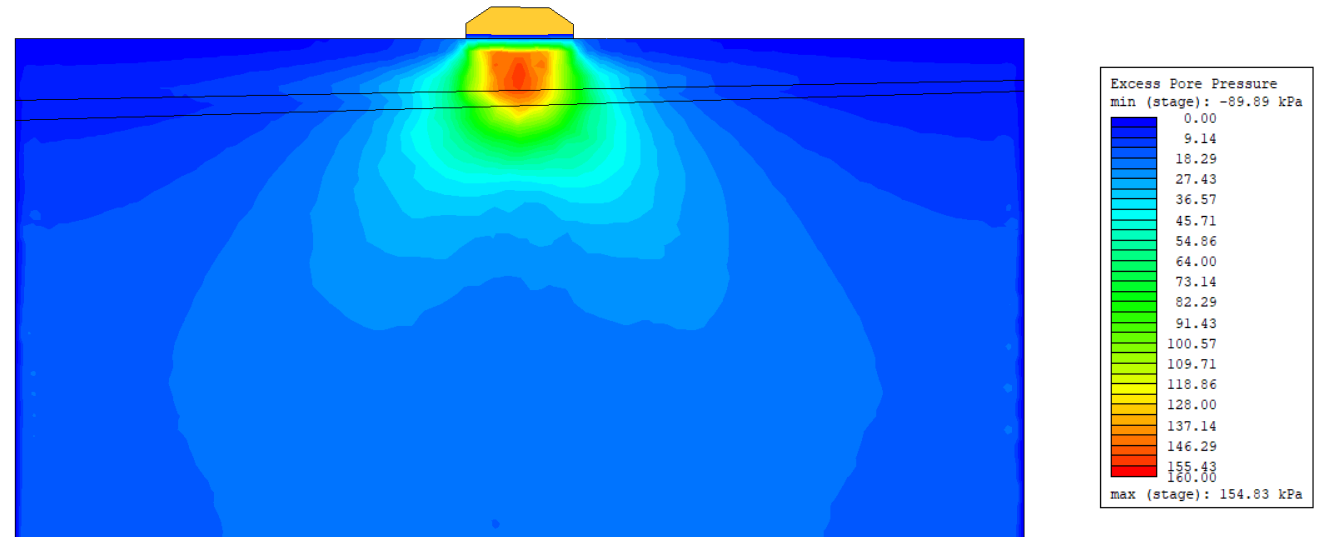
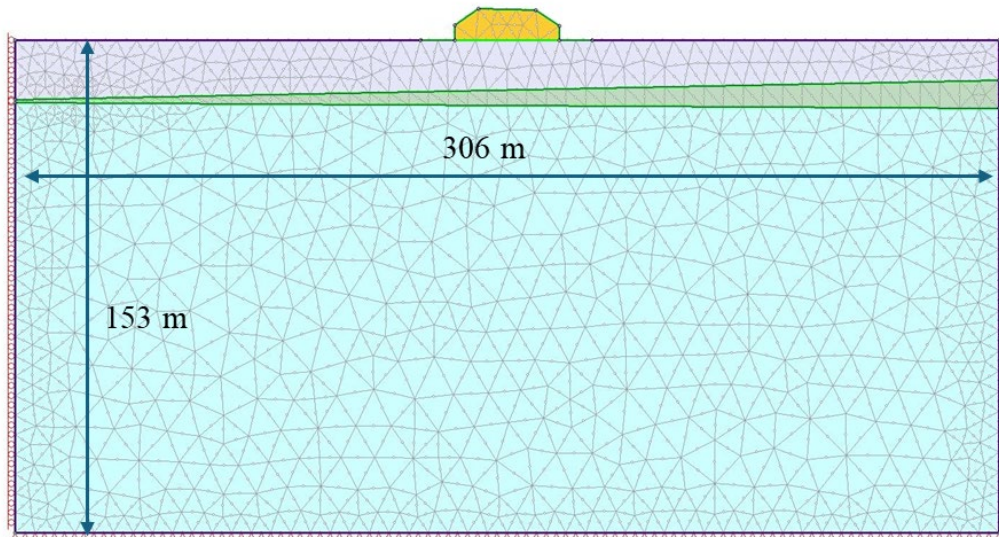
- Single Pile
- Circular flat footings
- Boreholes and vertical shaft excavations

- **Plane stress** conditions (rarely applied in geomechanics)



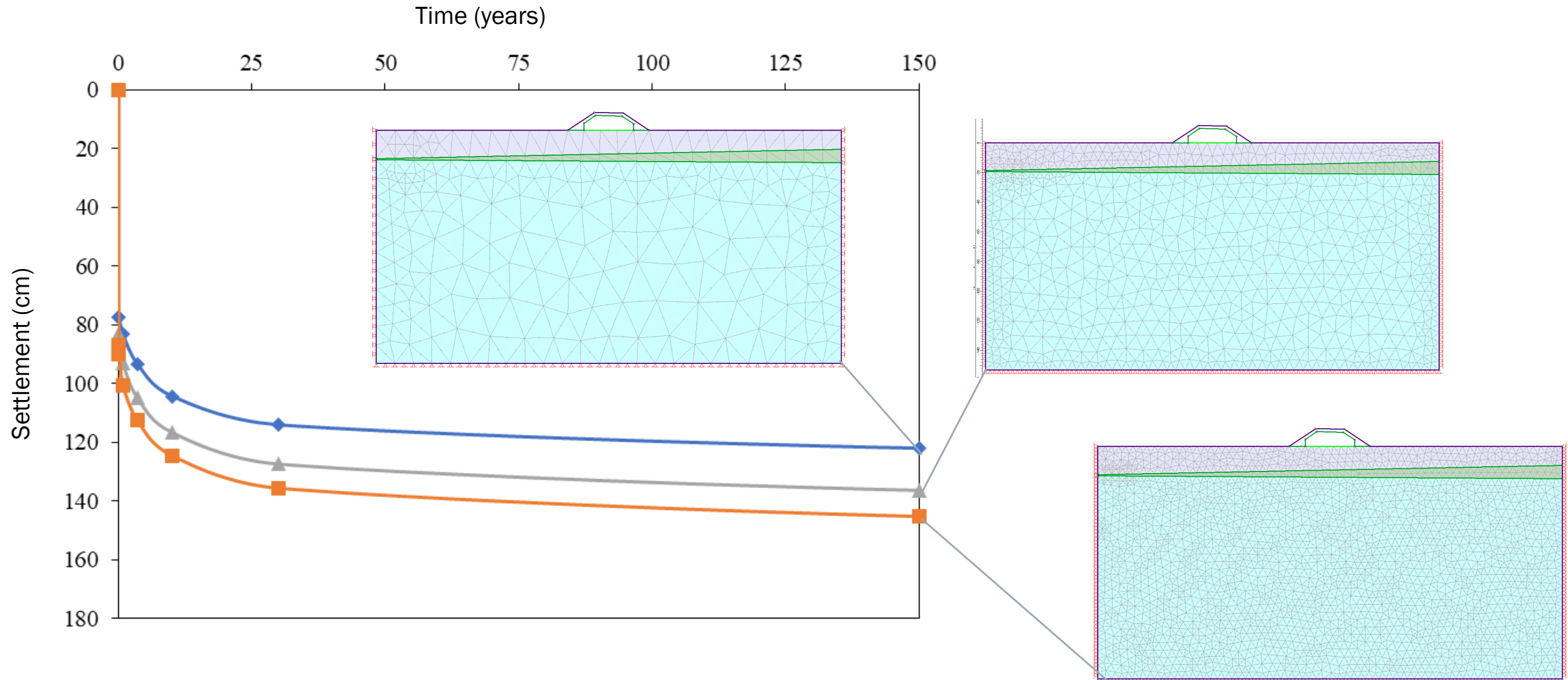
Model extension

- Boundaries have to be far enough so that they do not affect the stress and strain fields in the zone of interest for the analysis.



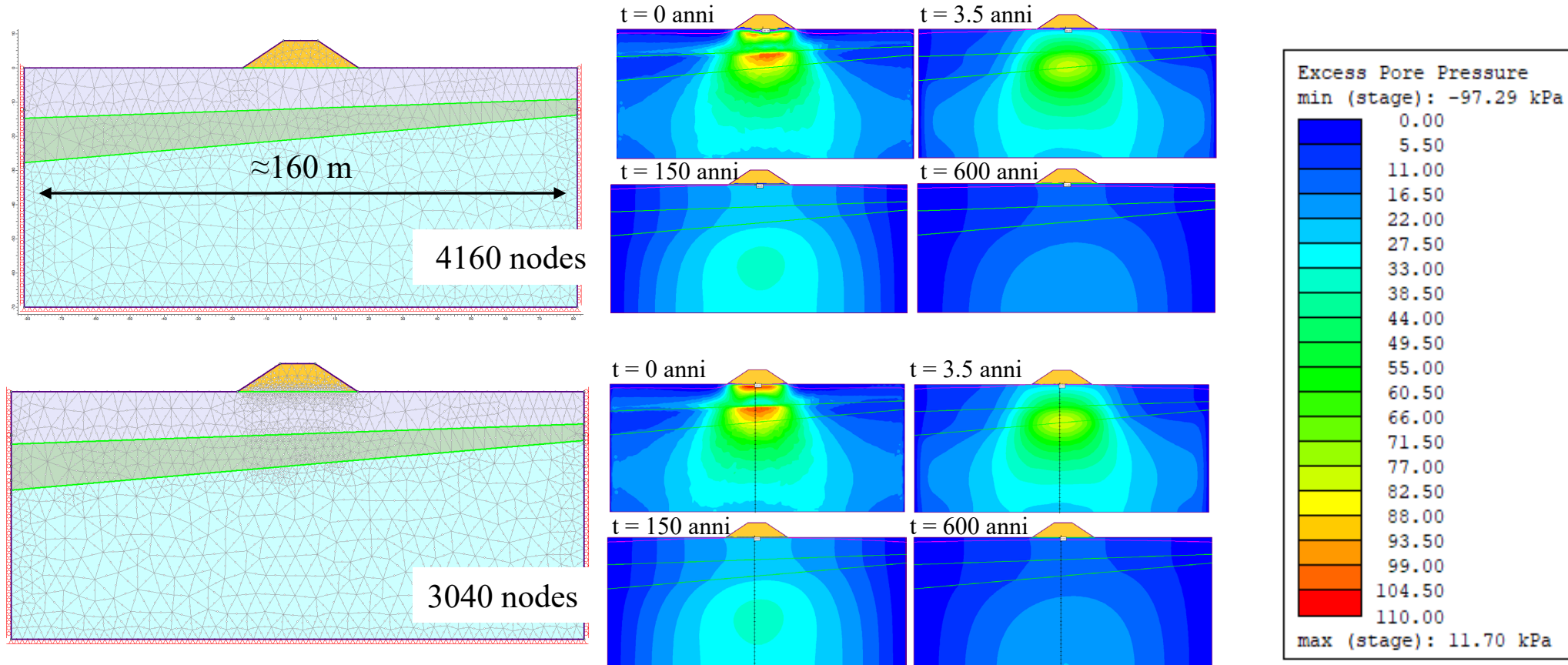
Discretization (meshing)

- Need to check that discretization is small enough, so that it doesn't influence significantly the results



Discretization (meshing)

- Need to check that discretization is small enough, so that it doesn't influence significantly the results

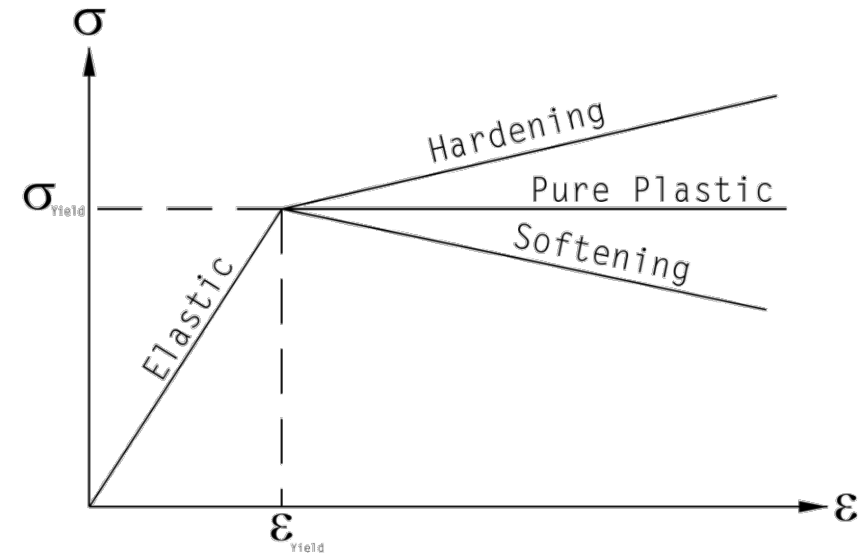


- Highly relevant when constitutive models with softening are used (localization of strains may occur). Regularization techniques may be needed.

Choice of the constitutive model

Dependent on:

- Material behavior
- Type and complexity of the problem
- Scale of the problem
- Expected range of deformations
- Accuracy requirements

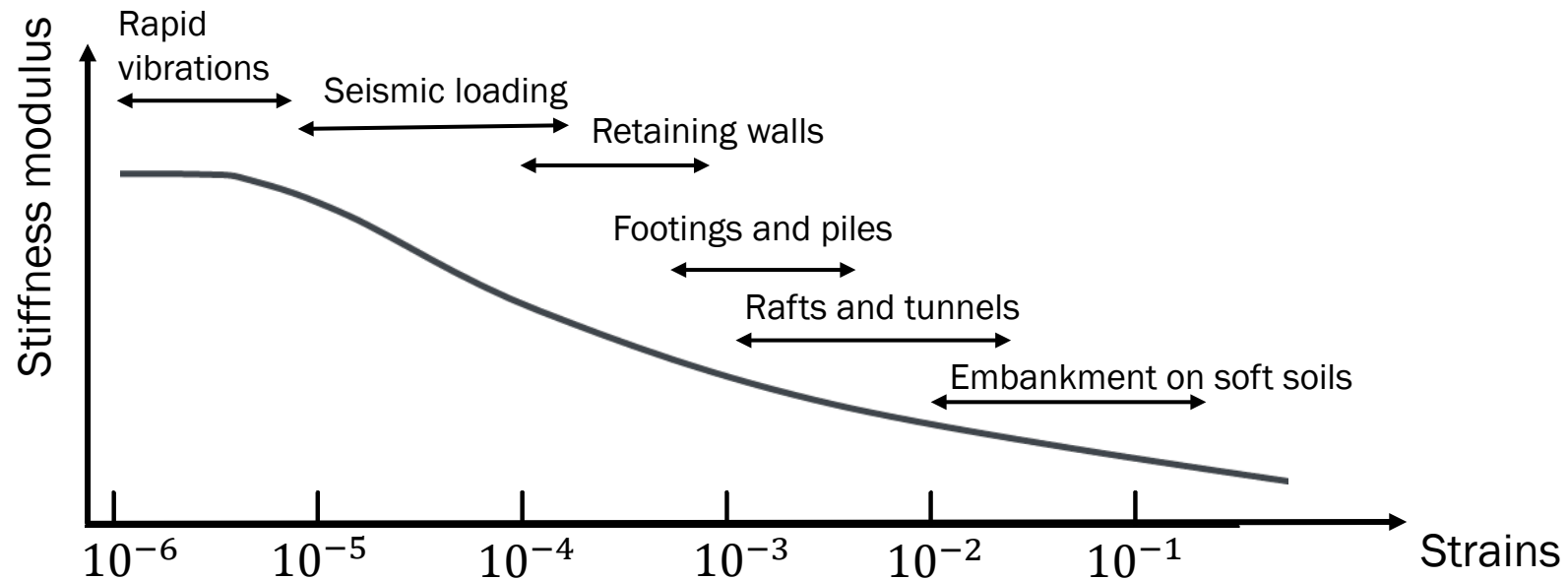


For certain works, due to the applied safety factors, elastic models can be sufficient.

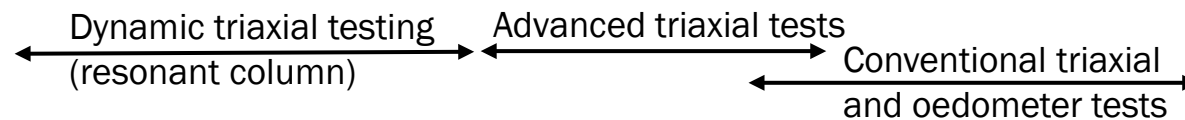
Elasto-plasticity is needed when the sequence of operations (e.g. excavations, installation of retaining structures, ...) is a key element

Determination of model parameters

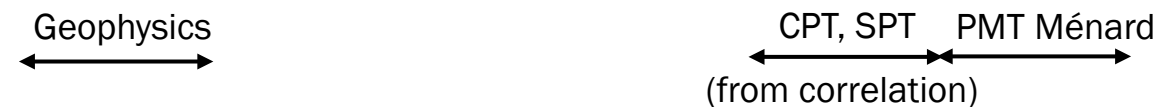
Laboratory and in-situ tests



Laboratory tests



In-situ tests



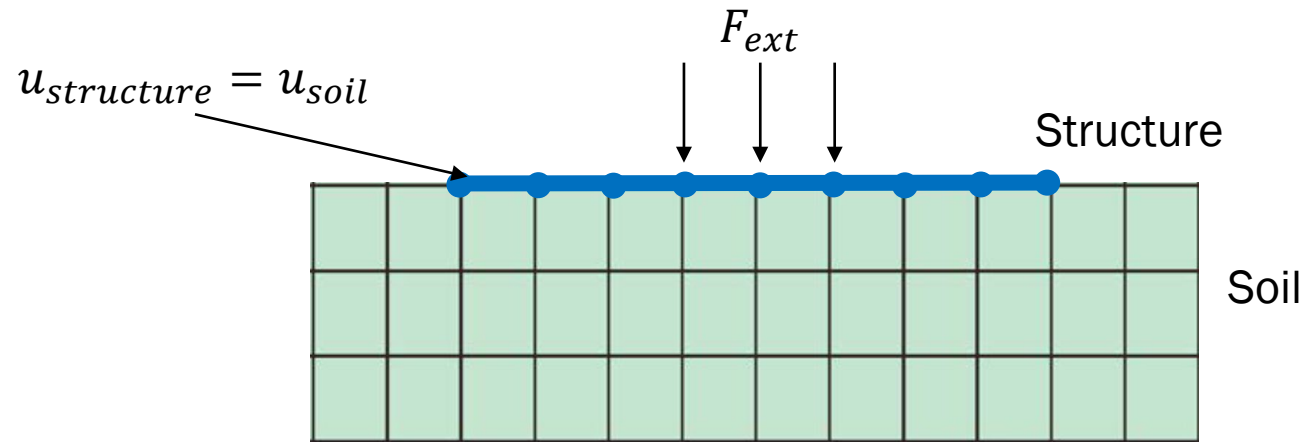
Soil-structure interactions: choice of the structural element type

- Beam elements
 - For structures that can be loaded in tension/compression, shear and moment
 - Navier-Bernoulli: neglect shear strain (thin elements)
 - Timoshenko: consider shear strain (thick elements)
 - Pile, retaining structure in 2D plane strain
- Plate or shell elements
 - For structures loaded that are susceptible to arching effects
 - As for beam elements, shear strain can be neglected for thin elements
 - Retaining structure, raft
- Membranes
 - For structures that can only be loaded under tension
 - Geomembranes

Soil-structure coupling

Direct coupling

- Shared mesh nodes that experience the same deformations



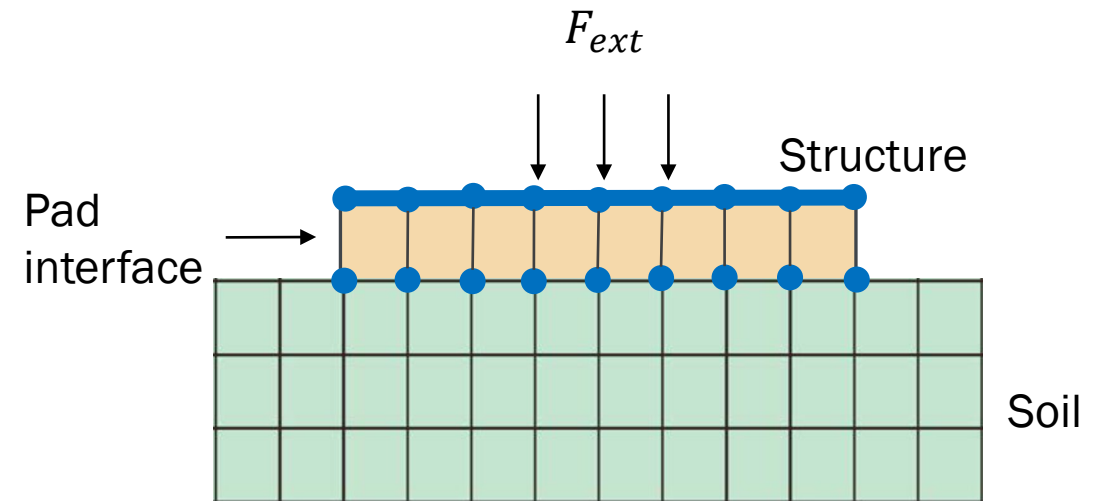
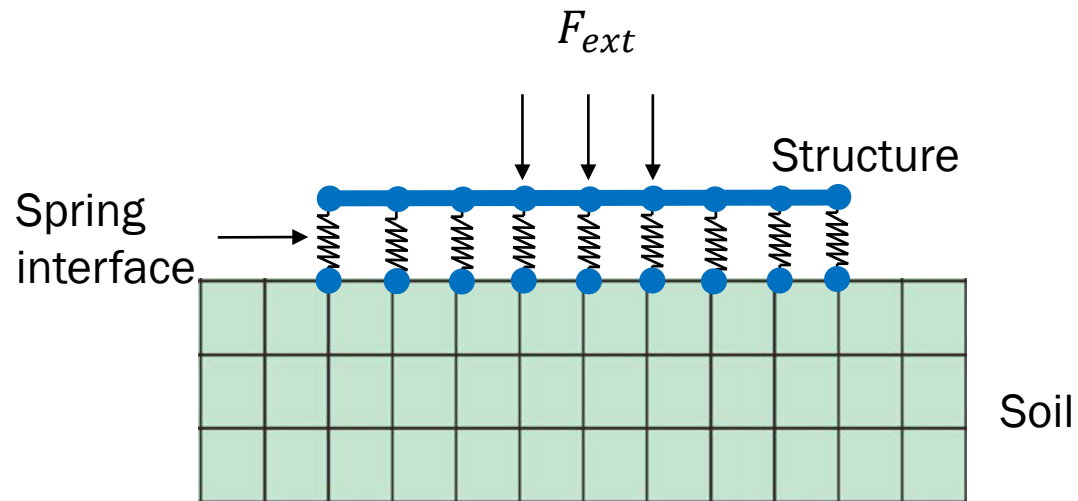
Easy and direct implementation

Limitation for non-linear effects between the structure and the soil
→ Implementation of an interface

Soil-structure coupling

Using an interface element

- Spring elements or pad elements are introduced to control the contact interface and simulate its nonlinear nature



Boundaries of the model

Typical boundary conditions of the model

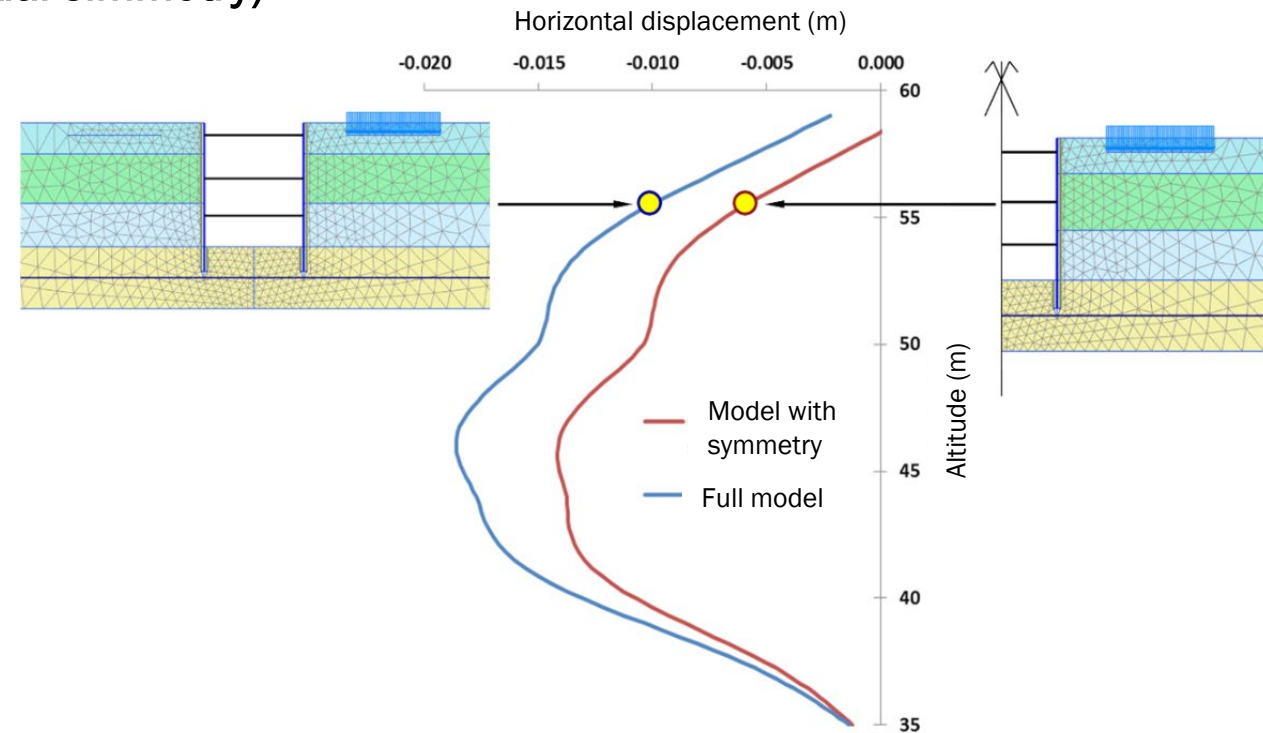
- Lateral : rollers, similar to symmetry boundary conditions
 - No normal displacements
- Bottom : fixed constraint
 - Displacement are zero in all direction
 - **Use of symmetry in 2D modelling (different from axial-simmetry)**



When you use the symmetry conditions, make sure this assumption is valid!

Depth of the model

- Insufficient depth can lead to underestimation of the settlement
- Significant depth can lead to overestimation of the settlement
 - Constant soil stiffness with depth may lead to unrealistic models
 - Non-linear evolution of the stiffness can improve the model (e.g. use of Modified Cam Clay)



Initial stress conditions

Initialization of the stress state is a very important step in numerical modelling

Goal: define a stress field according to various factors (dead weight of the soil, pore pressure, loading history, etc...)

K_0 method

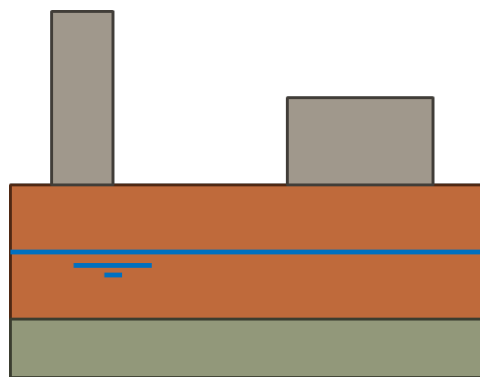
$$\sigma'_{h0} = K_0 \sigma'_{v0}$$

σ'_{h0} Initial horizontal effective stress

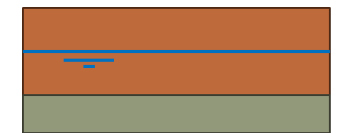
σ'_{v0} Initial vertical effective stress

K_0 Coefficient of lateral stress at rest

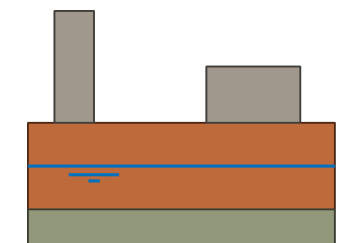
In case of existing buildings (or similar)



1. K_0 method (without any building)



2. Initial calculation step with the existing buildings

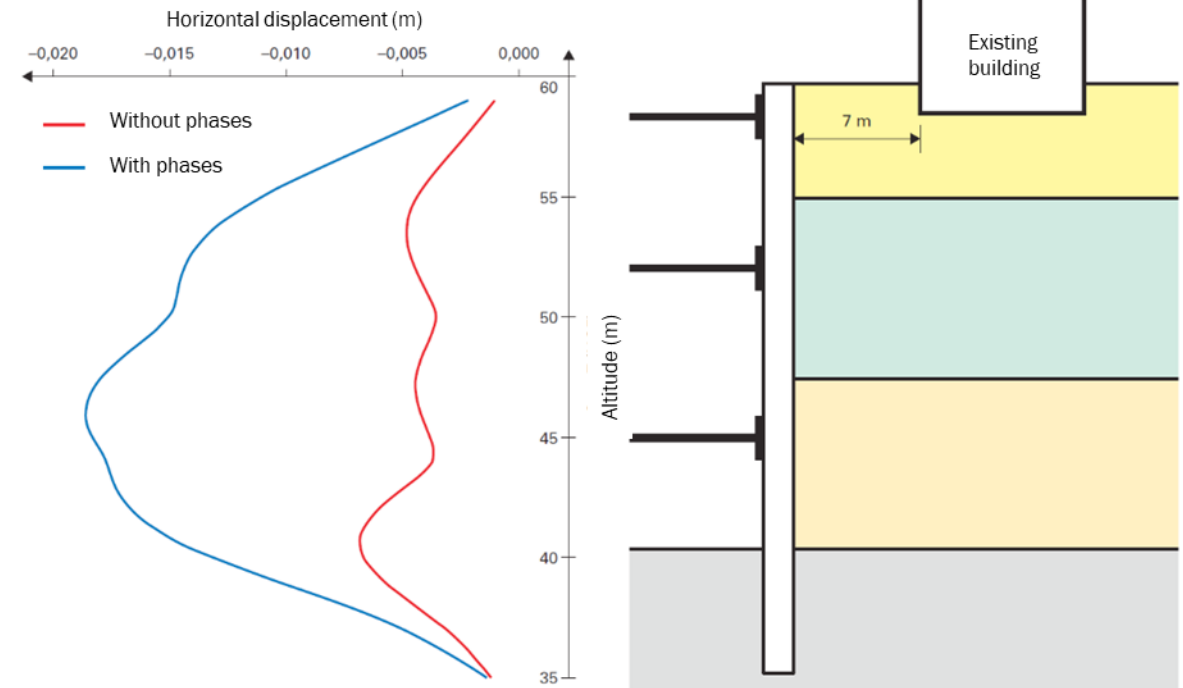


Construction phases

One of the main advantage of numerical modelling is the ability to model the loading history (prior and during the construction stage)

Omitting these aspects can lead to underestimation of the displacements and on the actions on the structural elements

An optimum and safe design does not only consider the final stage, but all the construction phases



Conclusion

The objectives must be defined prior to any step.

Although many situations can be simulated, the best choice is often not the most complex model.

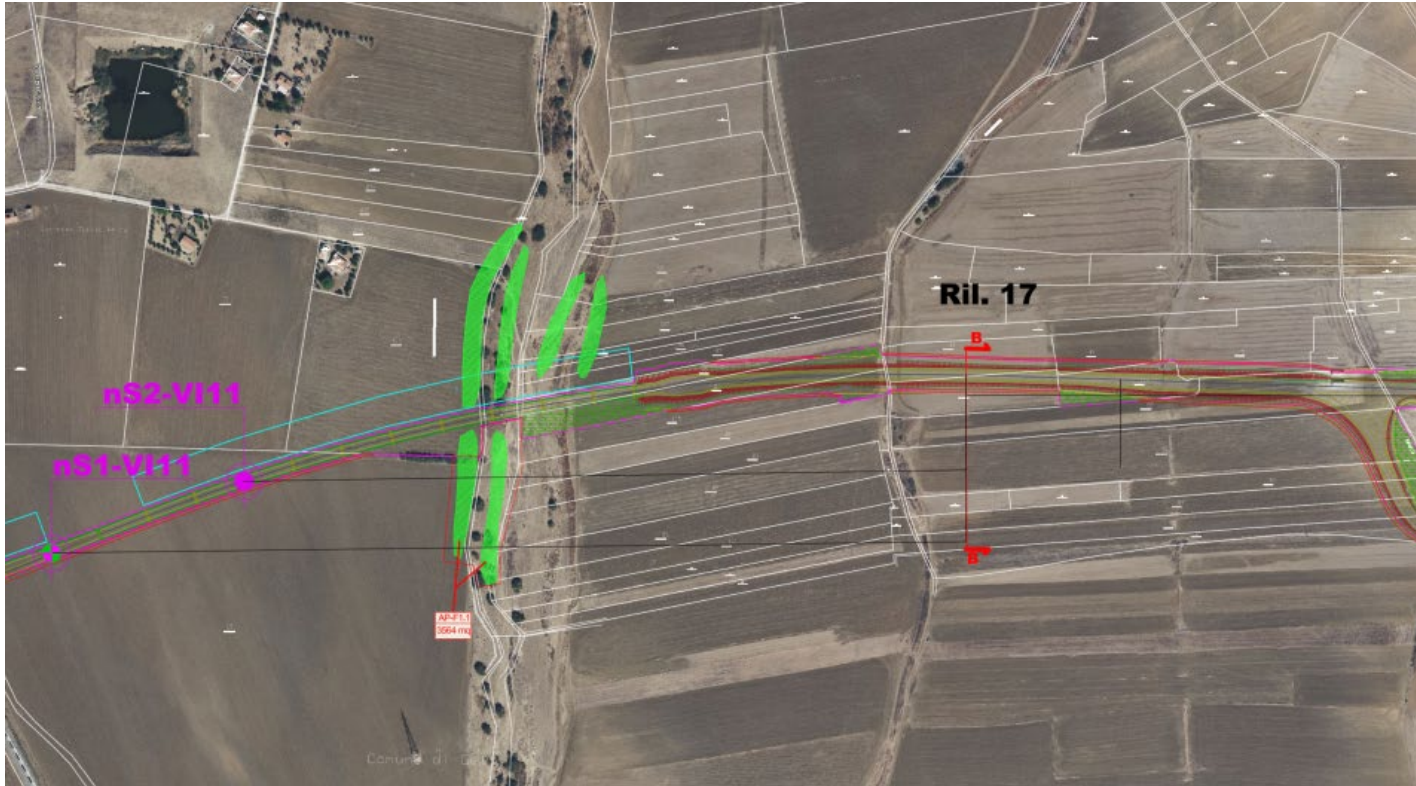
An excess of complexity can lead to numerical models that are difficult to control, especially from the large number of required parameters

A recommended approach is to start from simplified model (e.g., elasticity), and to progressively increase the complexity. This would allow a better control and comparison of the results.

Example 1

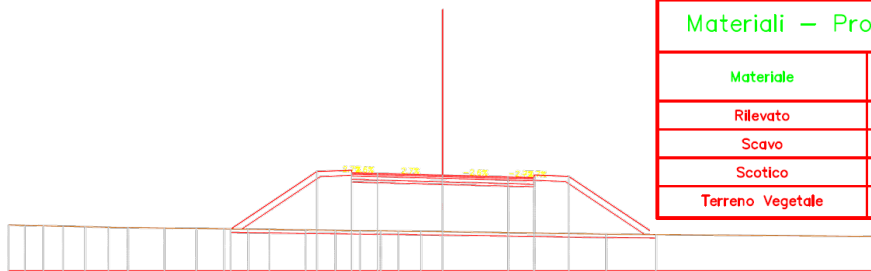
HM FORMULATION : SETTLEMENTS

Assessment of the effects of drainage systems on the settlement evolution of road embankments



Rilevato 17

Sez. 485
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 Scala : 1:200
 Q.Rif. : 20.00

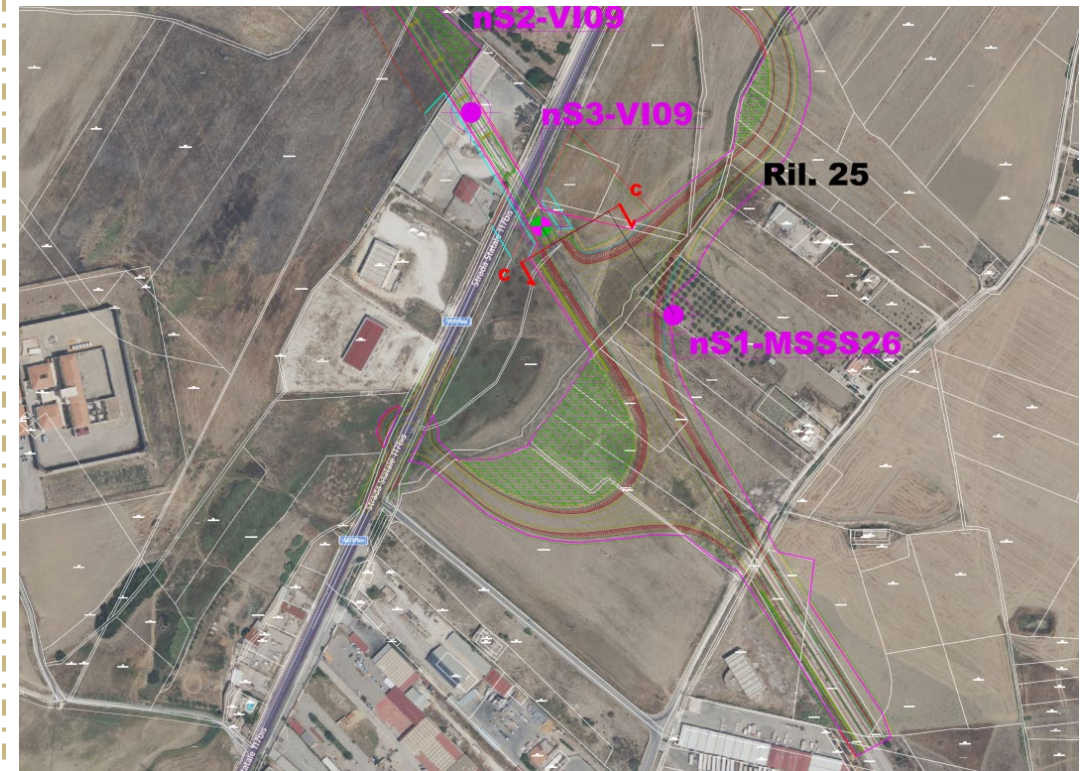
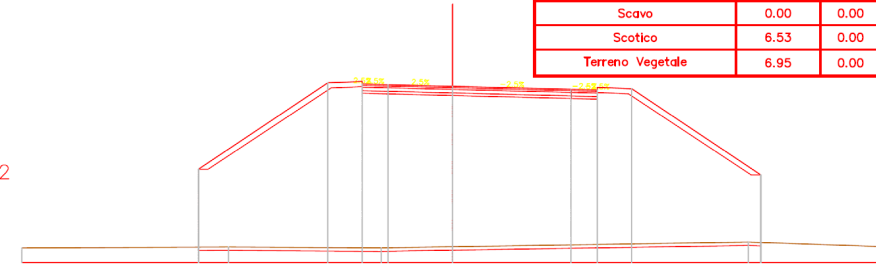


Materiali – Progr.9+680.00		
Materiale	Area (m ²)	Vol. (m ³)
Rilevato	53.29	1112.17
Scavo	0.00	0.00
Scotico	4.89	98.88
Terreno Vegetale	5.16	105.41

Rilevato 25

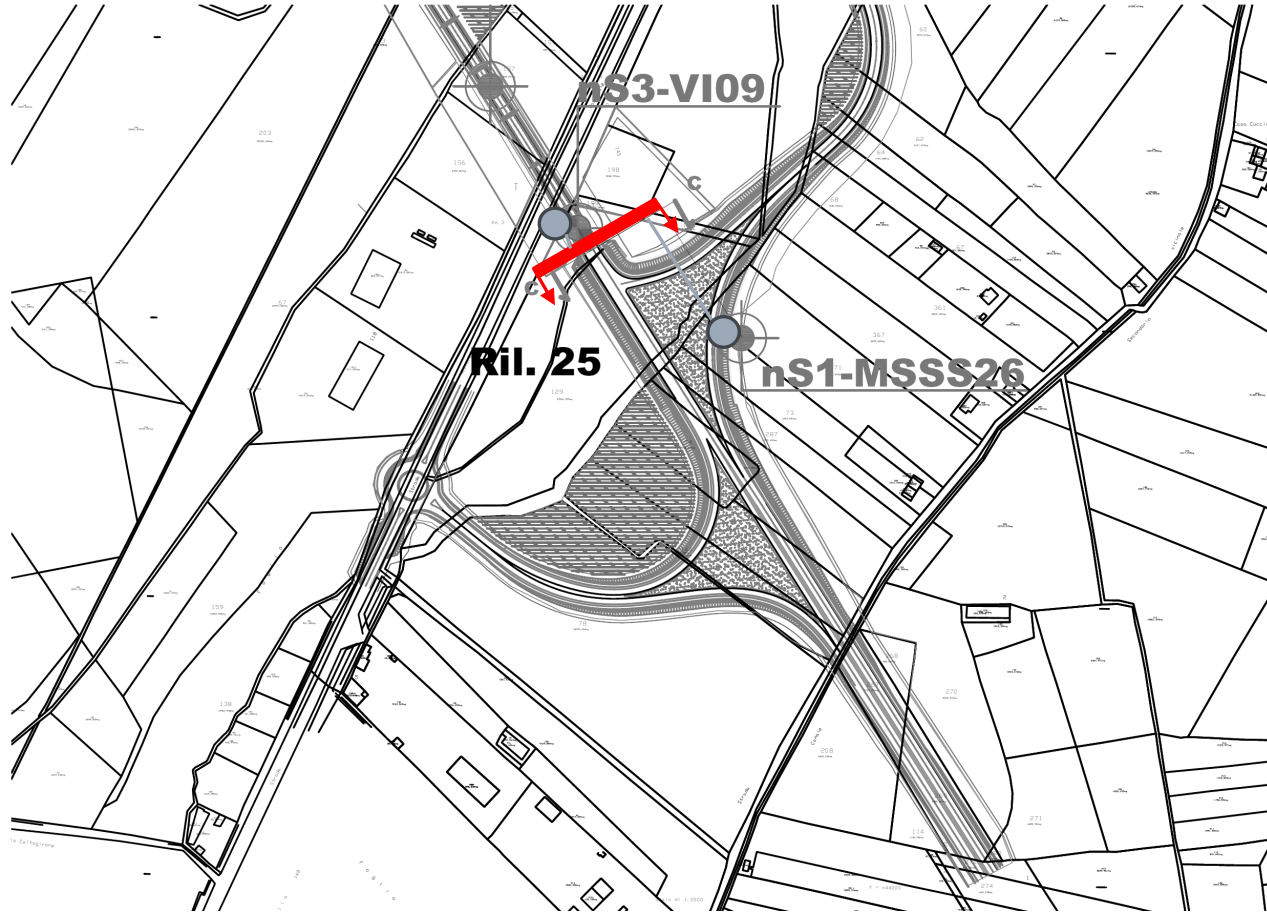
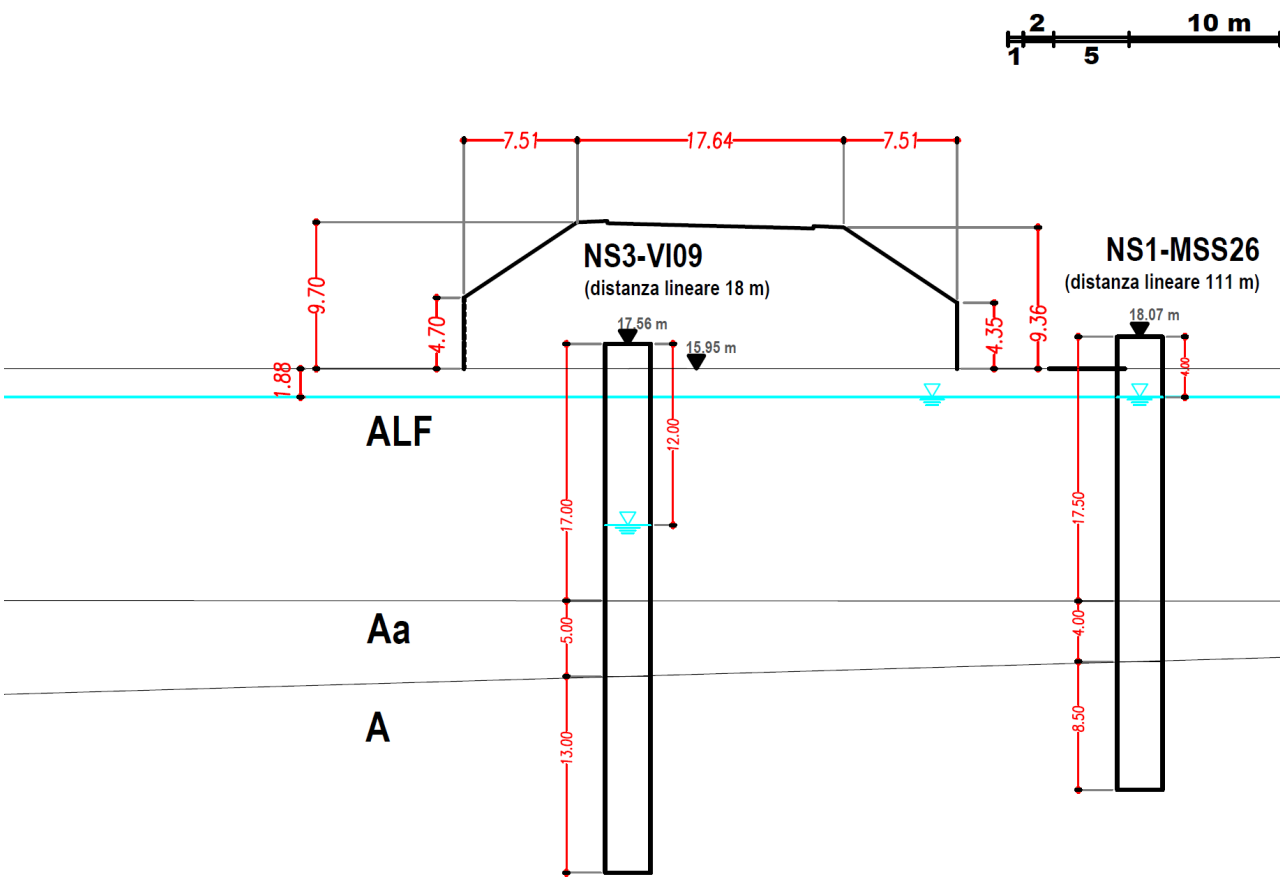
Sez. VI09_SPB
 Progr.: 15+285.22
 Scala : 1:200
 Q.Rif. : 15.00

Materiali – Progr.15+285.22		
Materiale	Area (m ²)	Vol. (m ³)
Rilevato	255.61	0.00
Scavo	0.00	0.00
Scotico	6.53	0.00
Terreno Vegetale	6.95	0.00



Assessment of the effects of drainage systems on the settlement evolution of road embankments

Ril. 25 - Sez. CC- Prog. 15+285



Description of the deposits involved

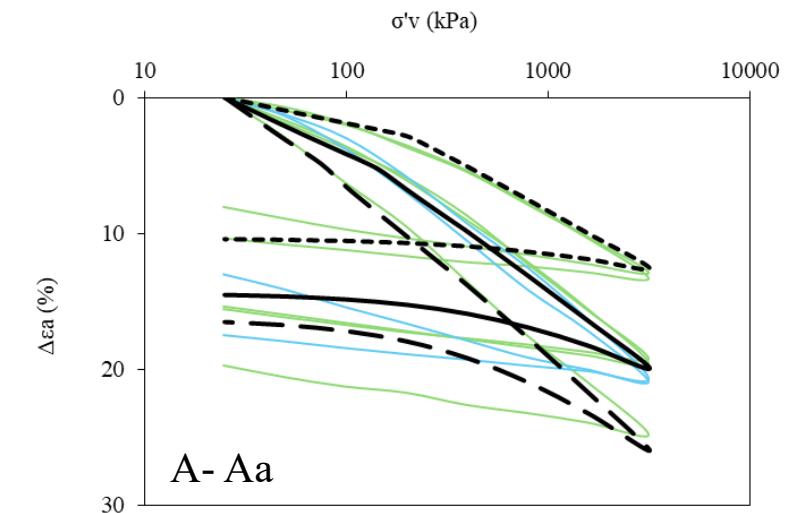
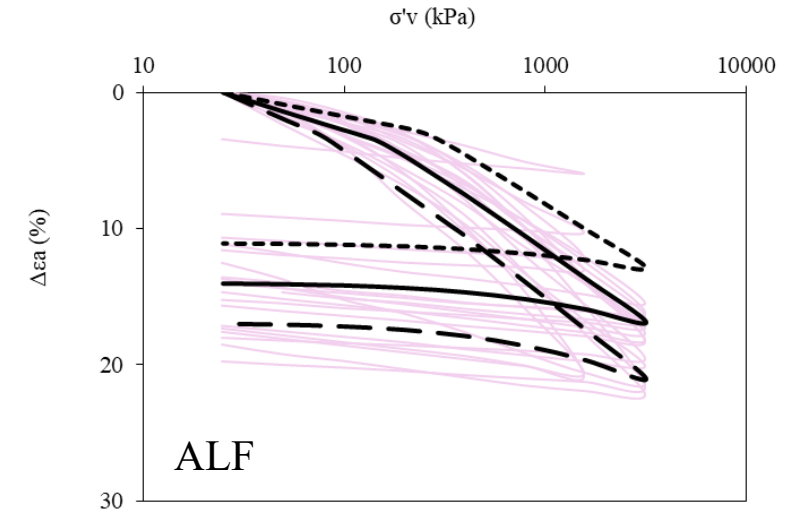
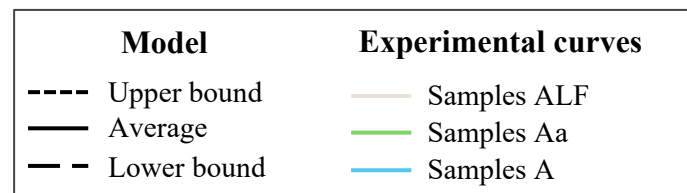
Deposit	Description
Fine-grained alluvial deposits (ALF)	Silty or sandy clay and clayey or sandy silt of variable colour and consistency. Presence of thin layers of sand or gravel.
Weathered clays (Aa)	Silt with clay or clayey silt of variable colour, with the presence of silty and sandy lenses. From firm to very firm.
Clays (A)	Silty or slightly sandy clay, grey-blue in colour, with possible dark grey, brown, or yellow coatings. From firm to hard.

Modified Cam Clay calibration

For each deposit, the set of parameters have been defined:

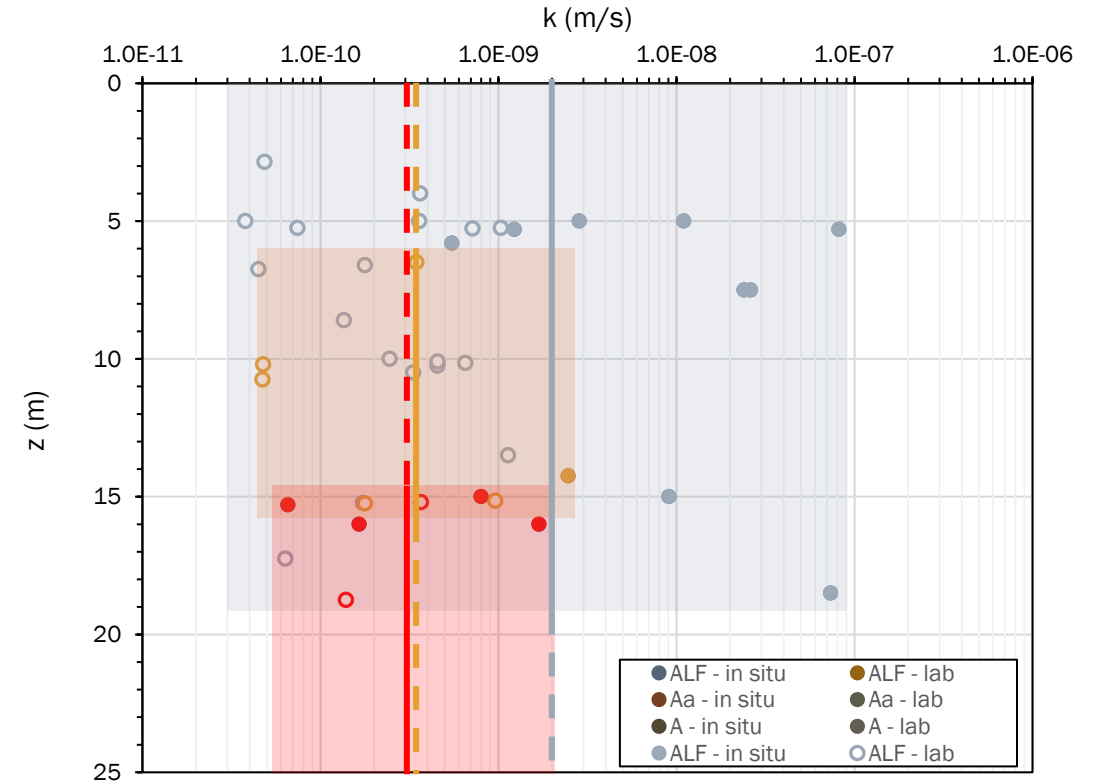
Parameters ALF	$k (-)$	$\lambda (-)$	$M (-)$	$p'_0 (kPa)$	$N (-)$	$\nu (-)$
Upper bound	0.0500	0.0650	0.772	112	1.834	0.3
Average trend	0.0300	0.0625	0.875	184	1.919	0.3
Lower bound	0.0200	0.0600	0.980	265	2.003	0.3

Parameters A e Aa	$k (-)$	$\lambda (-)$	$M (-)$	$p'_0 (kPa)$	$N (-)$	$\nu (-)$
Upper bound	0.0650	0.0750	0.898	96	1.915	0.3
Average trend	0.0425	0.0625	0.984	165	1.848	0.3
Lower bound	0.0200	0.0500	1.070	209	1.781	0.3



Permeability

Sample	Depth (m)	Deposit	σ_v (kPa)	c_v (m ² /s)	E_{oed} (kPa)	K (m/s)
NS1 VI03	13.5 - 14.0	ALF	196	2,2E-07	9129	2,3E-10
NS2 VI03	8.60 - 9.10	ALF	196	1,7E-06	14929	1,1E-09
NS2 VI04	10.0 - 10.5	ALF	196	1,1E-07	7942	1,4E-10
NS1 VI03	4.0 - 4.5	ALF	196	2,1E-07	8502	2,5E-10
NS3 VI09	10.5 - 11	ALF	196	4,3E-07	11661	3,6E-10
NS2 GA01	5.0 - 5.5	ALF	196	2,9E-07	8609	3,3E-10
NS2 MSS 08a	5.0 - 5.5	ALF	196	4,5E-08	11713	3,8E-11
NS1 MSS 06	5 - 5.5	ALF	196	2,3E-07	6252	3,6E-10
NS1 MSS 06	10 - 10.5	ALF	196	6,3E-08	8238	7,4E-11
NS1 MSS 06	15 - 15.45	ALF	196	4,4E-07	9439	4,5E-10
NS1 MSS27	5 - 5.5	ALF	196	2,2E-07	12370	1,7E-10
NS1 MSS27	10 - 10.4	ALF	196	9,1E-07	8594	1,0E-09
NS1 MSS09	5 - 5.5	ALF	196	7,2E-07	15519	4,6E-10
NS1 MSS09	10 - 10.3	ALF	196	6,7E-07	9150	7,2E-10
NS1 GA01	2.7 - 3	ALF	196	8,8E-07	13150	6,5E-10
NS2 VI11	17 - 17.5	ALF	196	6,5E-08	13152	4,9E-11
NS1 VI06	6.5 - 6.7	ALF	196	7,5E-08	11559	6,4E-11
NS1 VI09	6.5 - 7	ALF	196	1,8E-07	9716	1,8E-10
NS1 VI05	6.5 - 7	ALF	196	5,8E-08	12716	4,5E-11
NS1 VI04	6.5 - 7.0	Aa	196	2,2E-07	6100	3,5E-10
NS2 MSS 08a	10 - 10.4	Aa	196	5,4E-08	11108	4,8E-11
NS1 MSS 27	15- 15.5	Aa	196	2,2E-07	12183	1,8E-10
NS1 MSS09B	15- 15.3	Aa	196	8,0E-07	8183	9,6E-10
NS2 VI09	10.5 - 11	Aa	196	6,9E-08	14216	4,7E-11
NS2 MSS 08a	15 - 15.4	A	196	1,8E-07	4899	3,7E-10
NS2 VI07	18,5 - 19	A	196	1,2E-07	8237	1,4E-10



Assumed:

ALF: $k = 3 \times 10^{-11} \div 9 \times 10^{-8}$ 2×10^{-9}

Aa: $k = 5 \times 10^{-11} \div 3 \times 10^{-9}$ 3×10^{-10}

A: $k = 7 \times 10^{-11} \div 2 \times 10^{-9}$ 3×10^{-10}

Modelling of the embankments with RS2

For modelling the embankments with RS2, the «average trend» set of parameters for each deposit has been used:

The following tables show the adopted constitutive parameters and geotechnical properties for each deposit:

Formazione	K (m/s)	γ (kN/m ³)	n_0 (-)	K_0
ALF	2e-09	19.2	0.408	0.62
Aa	3e-10	19.4	0.415	0.58
A	3e-10	19.4	0.415	0.58

- K is the permeability
- γ is the weight per unit volume (average value of the samples)
- n_0 is the initial porosity (average value of the samples)
- K_0 is the ratio between horizontal and vertical effective stresses; estimated from the values of φ'

Formazione	k (-)	λ (-)	M (-)	p'_0 (kPa)	N (-)	ν (-)
ALF	0.0300	0.0625	0.875	184	1.919	0.3
Aa	0.0425	0.0625	0.984	165	1.848	0.3
A	0.0425	0.0625	0.984	165	1.848	0.3

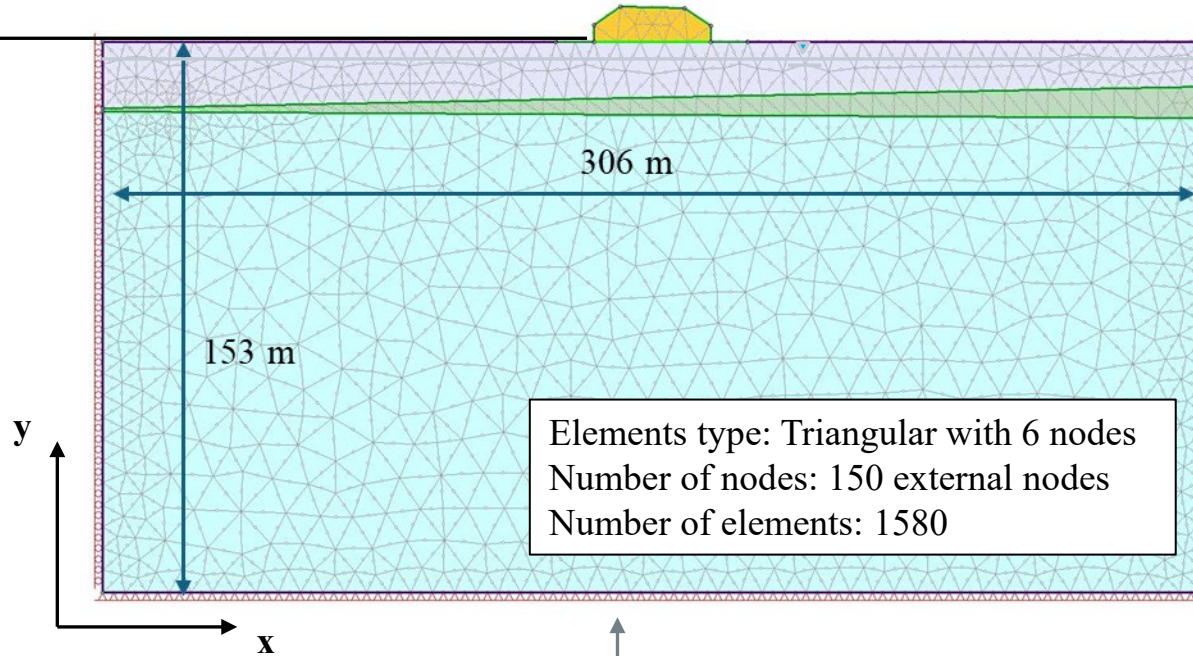
For the embankment, the Mohr – Coulomb constitutive model was adopted; The constitutive parameters and geomechanical properties are listed below:

Formazione	φ' (kPa)	E (kPa)	ν (-)	n_0 (-)	c' (kPa)	γ (kN/m ³)	K (m/s)
Rilevato	35	40000	0.3	0.5	10.5	20	$1 \cdot 10^{-7}$

- E is the Young's modulus
- φ' is the shear strength angle
- c' is the intercept cohesion

Embankment 25

Reference plane



Elements type: Triangular with 6 nodes
Number of nodes: 150 external nodes
Number of elements: 1580

$h = -1.88 \text{ m}$
 $u_x = 0$
 $u_y = \text{allowed}$

$\frac{\partial h}{\partial n} = 0$
 $u_x = u_y = 0$

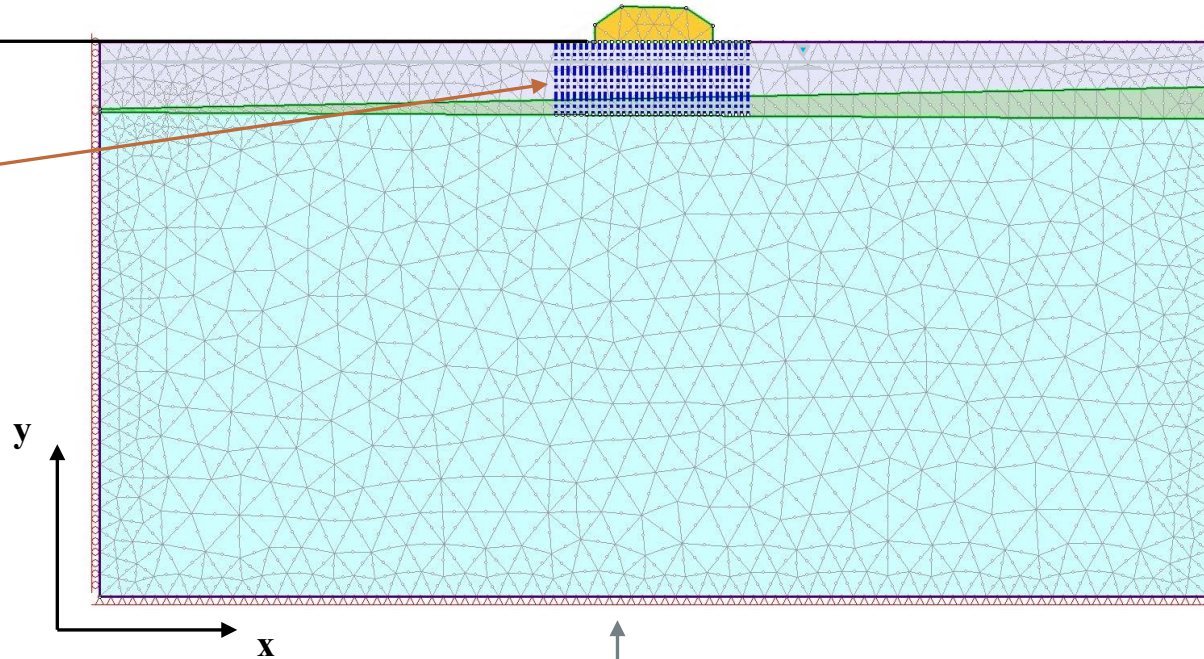
Embankment 25 with drains

Reference plane

Drains

- Theoretical diameter equal to 66 mm;
- Length equal to 20 m;
- Spacing arrangement of 1.8 m (2D)

Initial and final steady-state hydraulic condition



$$h = -1.88 \text{ m}$$

$$u_x = 0$$

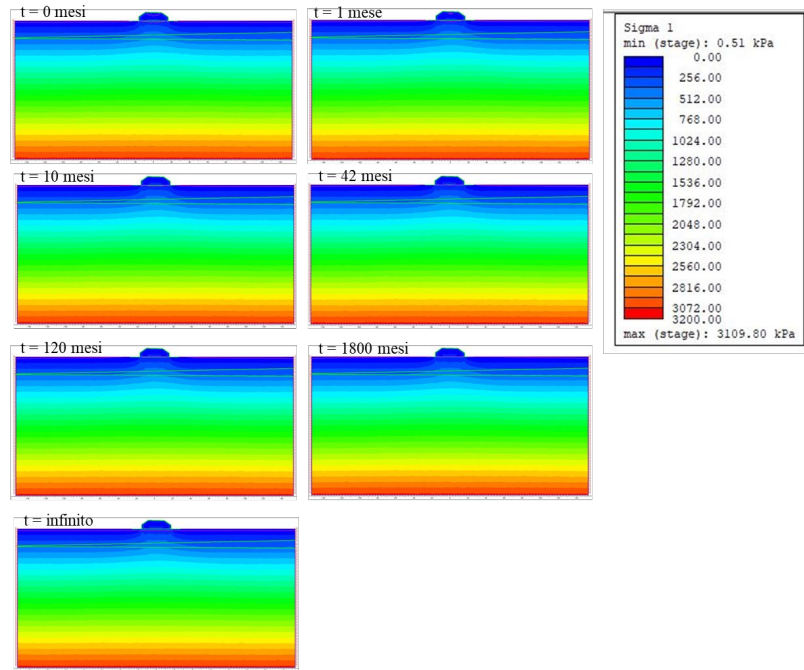
$$u_y = \text{allowed}$$

Elements type: Triangular with 6 nodes
 Number of nodes: 150 external nodes
 Number of elements: 1580

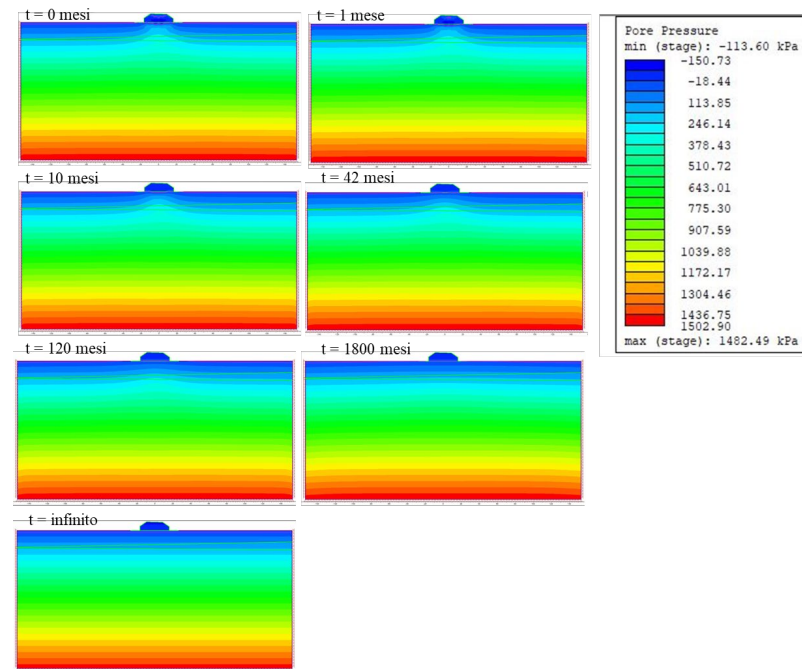
$$\frac{\partial h}{\partial n} = 0$$

$$u_x = u_y = 0$$

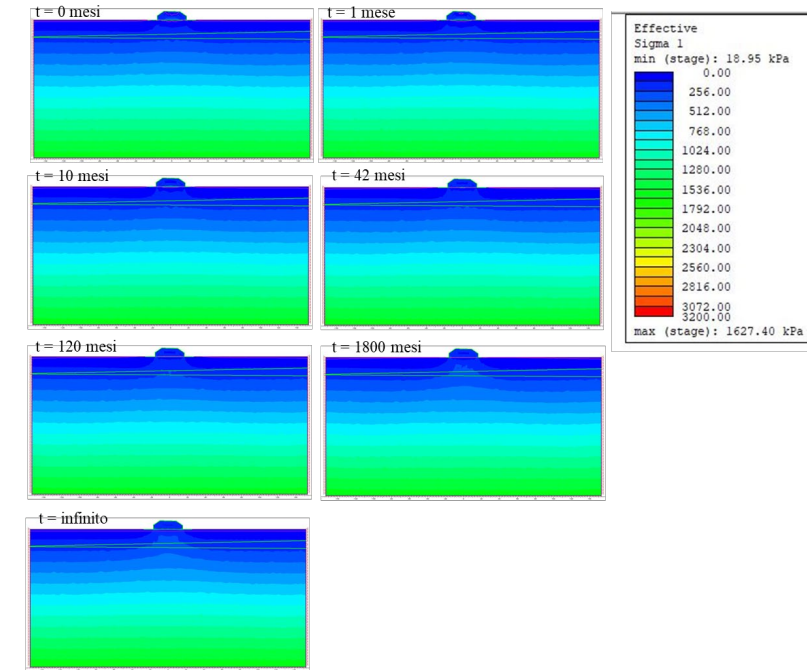
Evolution of vertical total stress



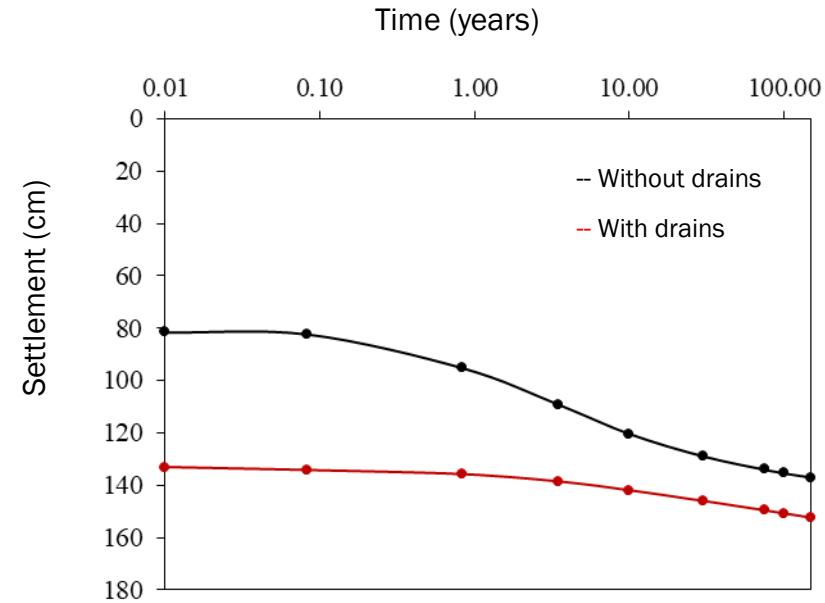
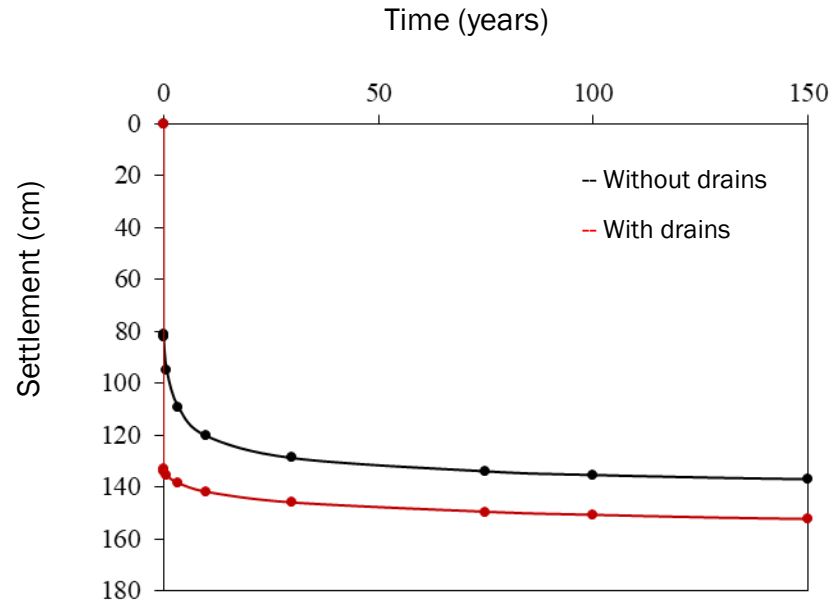
Evolution of pore water pressure



Evolution of vertical effective stress



Embankment 25



Time	Settlement without drains (cm)	Settlement with drains (cm)	Embankment (8 m) settlements with drains from PD (cm)	Embankment (10 m) settlements with drains from PD (cm)
0+	81.4	133.1	-	-
1 month	82.2	134.2	-	-
8 months	-	-	79	122
10 months	95.1	135.8	-	-
42 months	109.2	138.6	82	124
10 years	120.3	141.9	-	-
30 years	128.9	146.1	-	-
75 years	134.0	149.6	-	-
100 years	135.4	150.8	-	-
150 years	137.1	152.5	-	-
Infinite time	162.9	164.2	98	139

Example 2

HM FORMULATION : LANDSLIDE

Hydrogeological and geomechanical modelling of the Steinernase landslide (Switzerland)

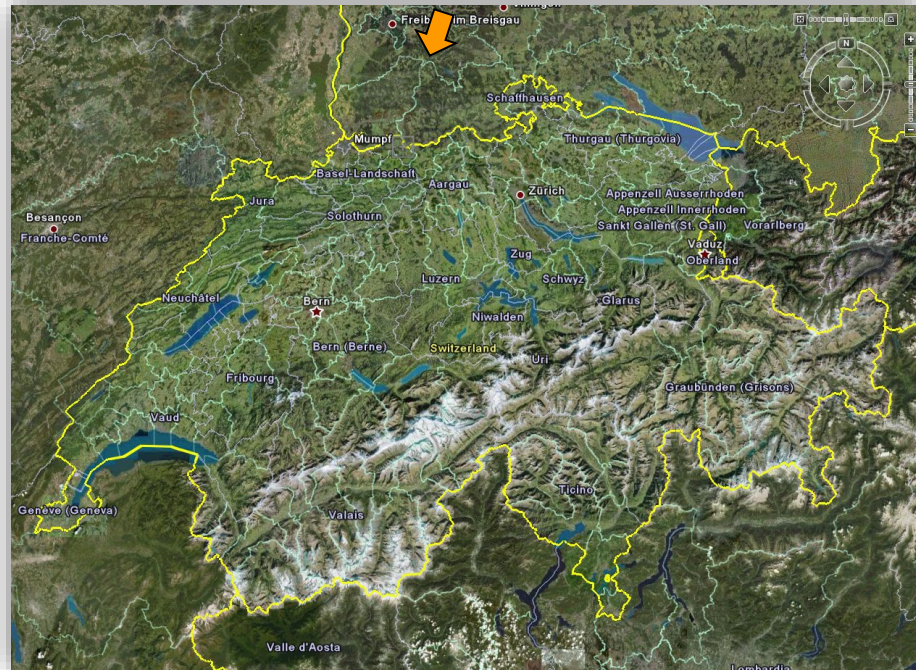


Image by Google Earth

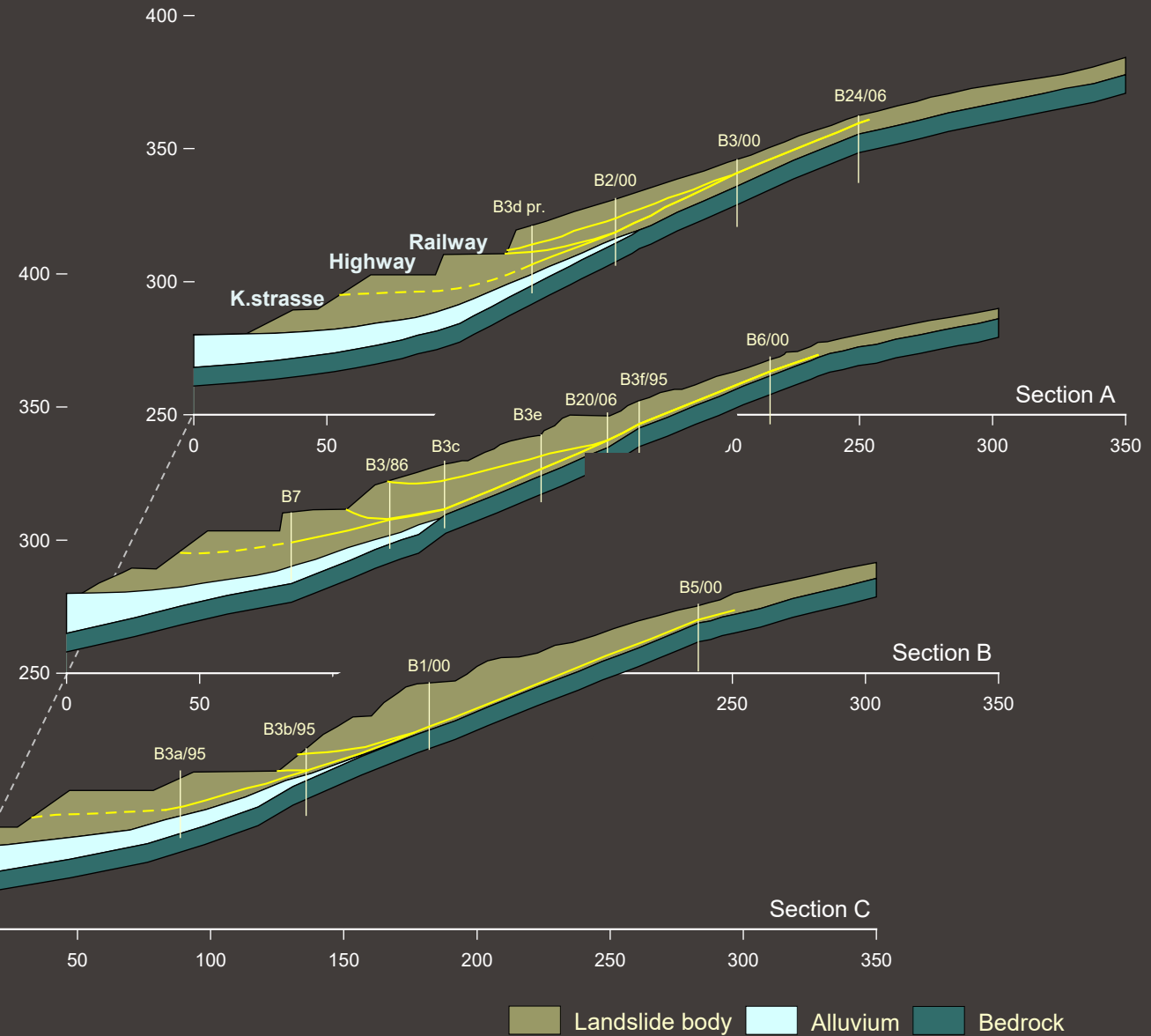
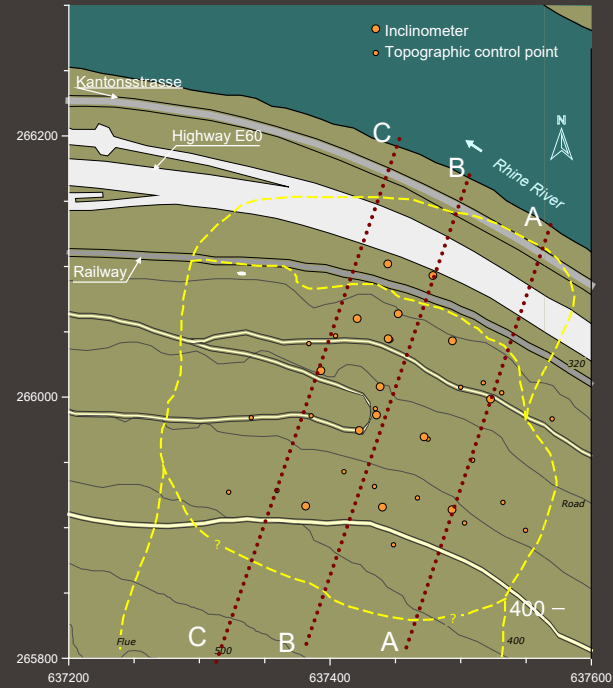




Main features

- Active area: $\sim 1 \text{ km}^2$
- Mean inclination: 20°
- Mean depth: 10 - 15 m
- Movement rates: 1-2 cm/y

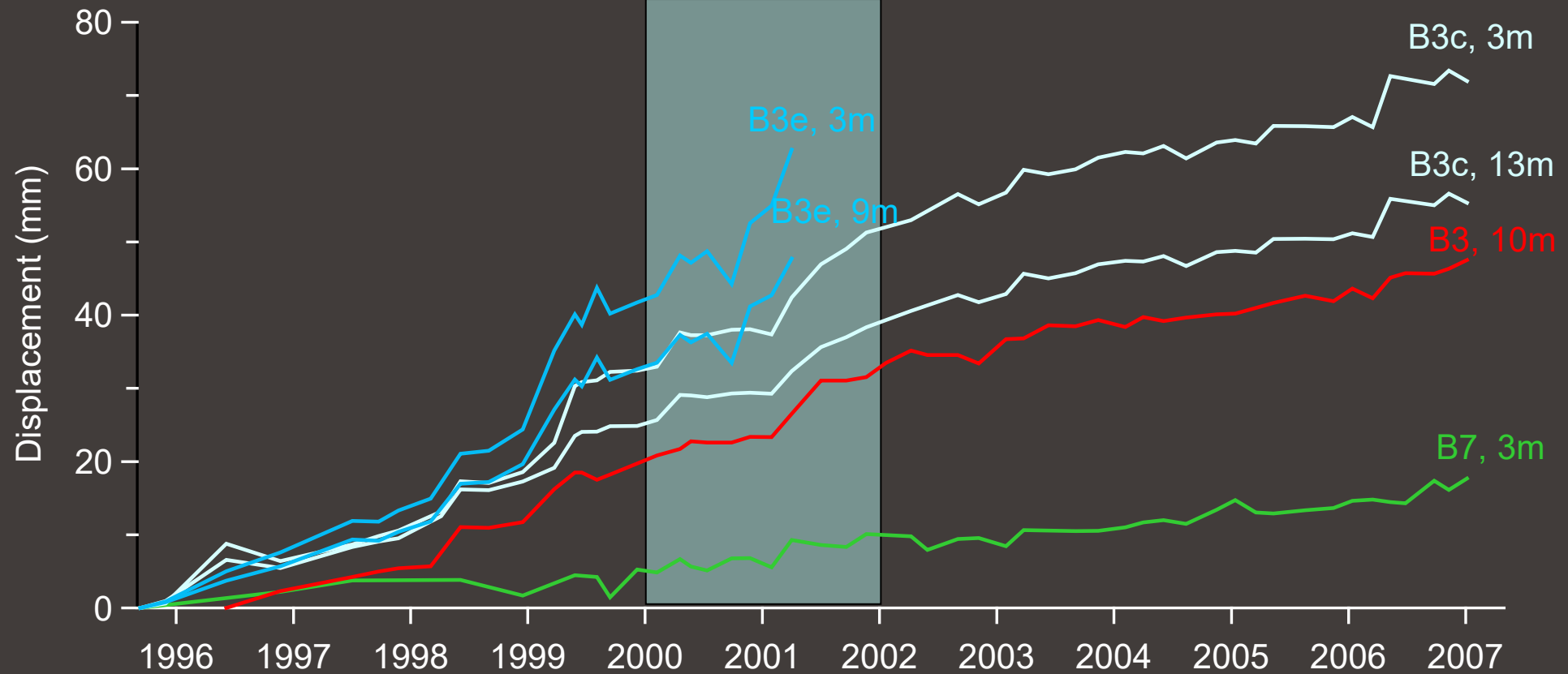
Analysis of the multi-surface mechanism

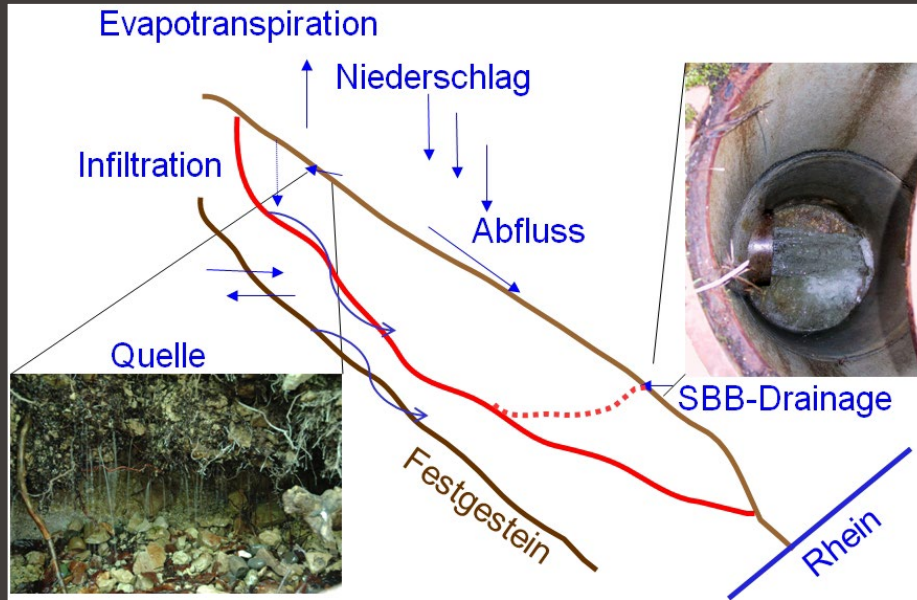


The shape of the different failure surfaces has been identified in different 2D sections combining all available inclinometer readings for the period 1985 - 2006

Model calibration and validation

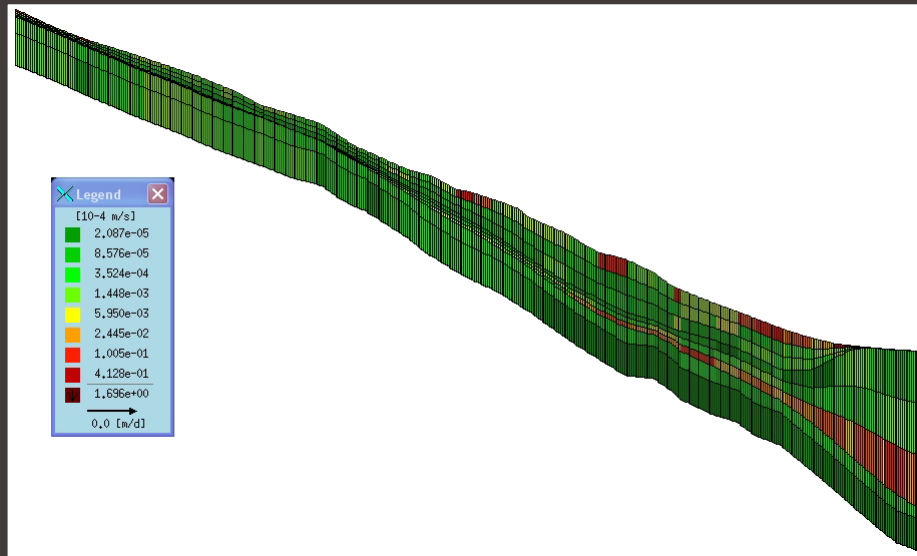
Hydrological model calibration for the period 2000 - 2001



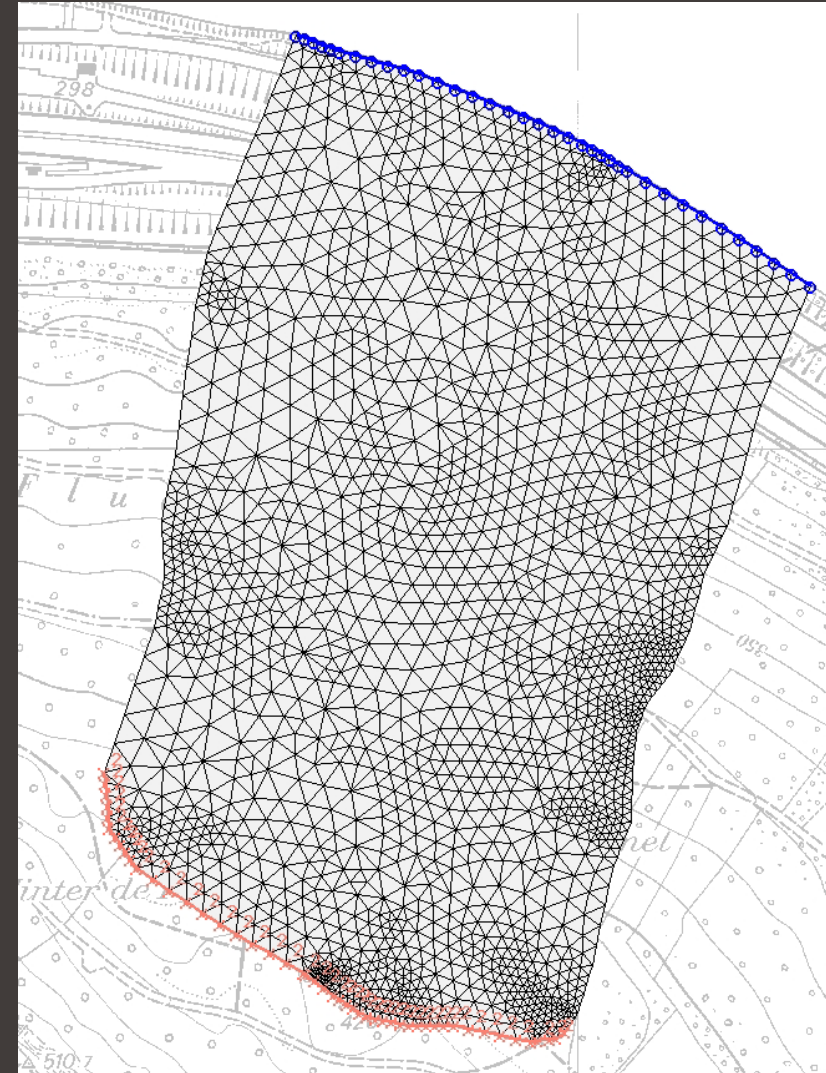


The hydro-geological model

Laurent Tacher and Daniel Locher (Geolep/EPFL)



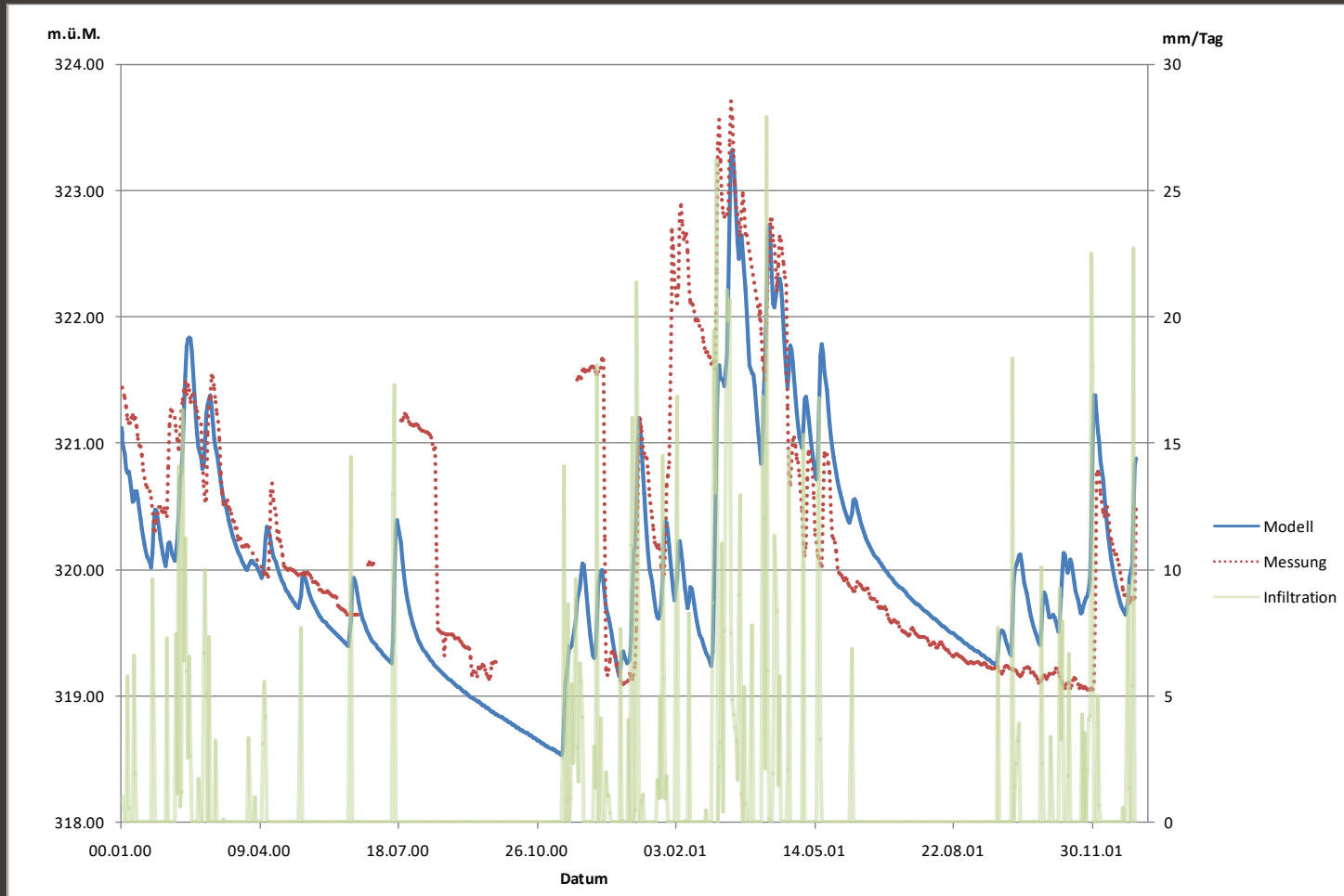
Cross section showing layers with different permeability values



Planar view of the 3D hydro-geological model

The hydro-geological model

Geolep/EPFL



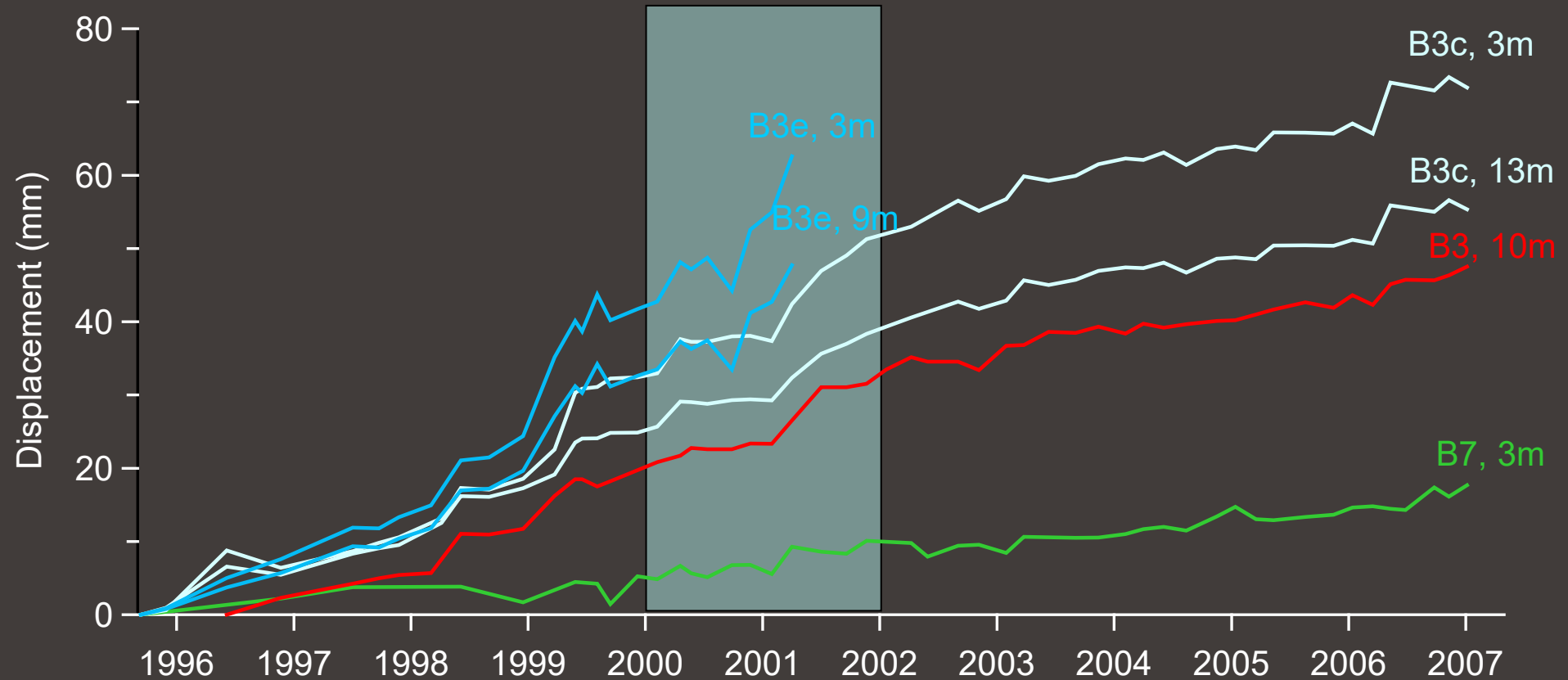
Comparison between the measured and the computed total head at B3c
(2000 – 2001)

Model calibration and validation

Hydrological model calibration for the period 2000 - 2001



Pore water pressure distribution are obtained as an output of the calibration



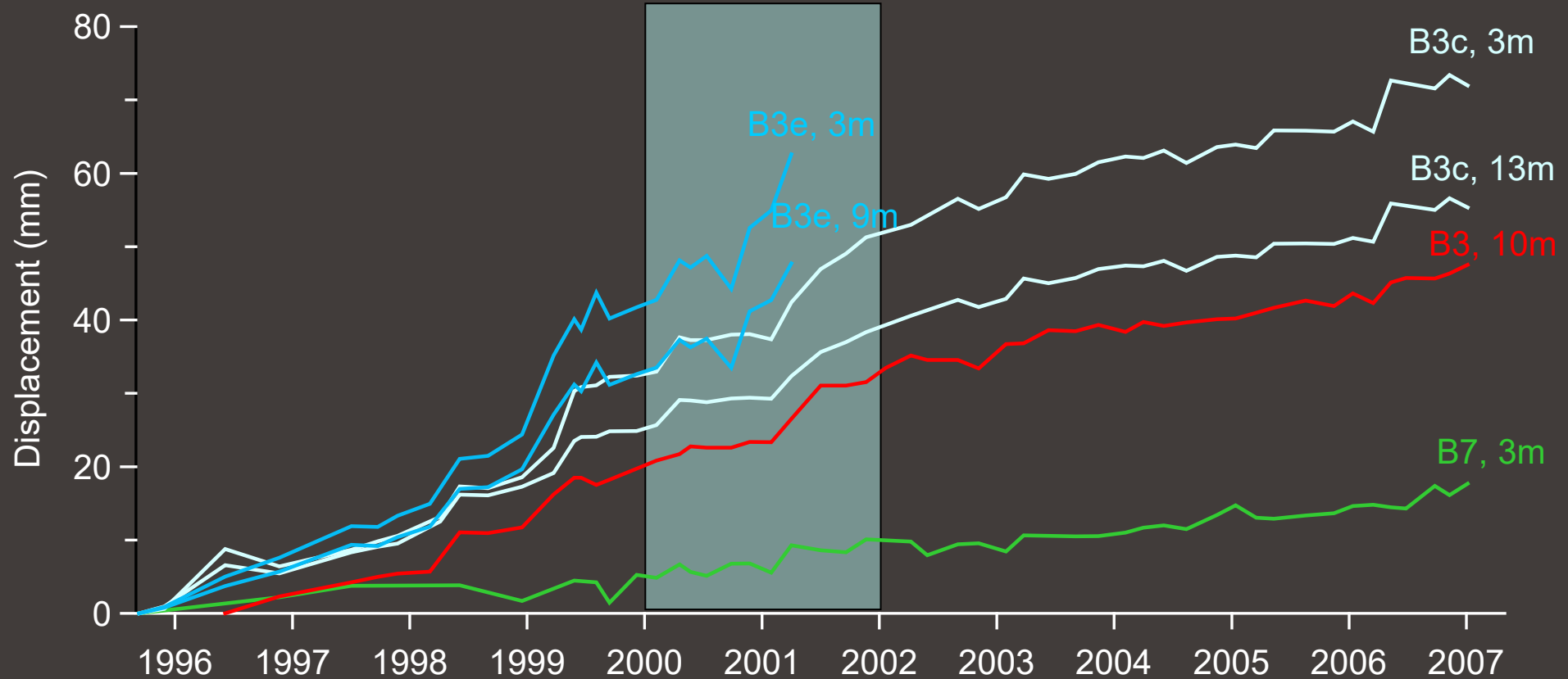
Model calibration and validation

Hydrological model calibration for the period 2000 - 2001

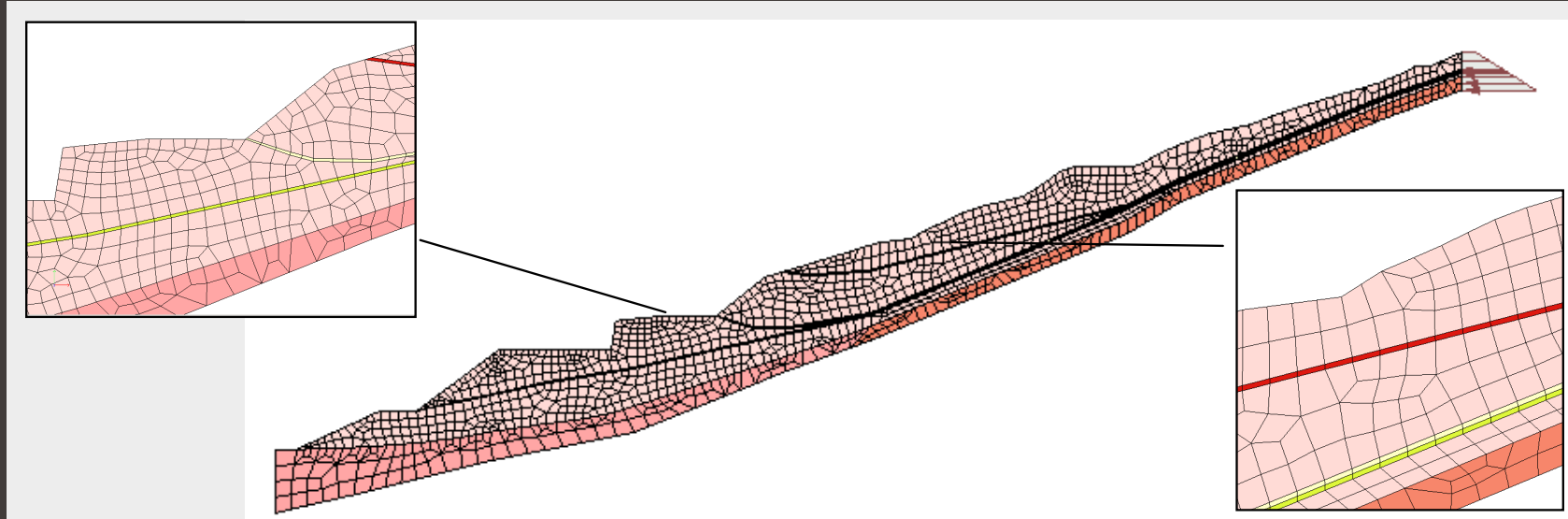


Pore water pressure distribution are obtained as an output of the calibration

Geomechanical model calibration for the period 2000-2001



Geomechanical model



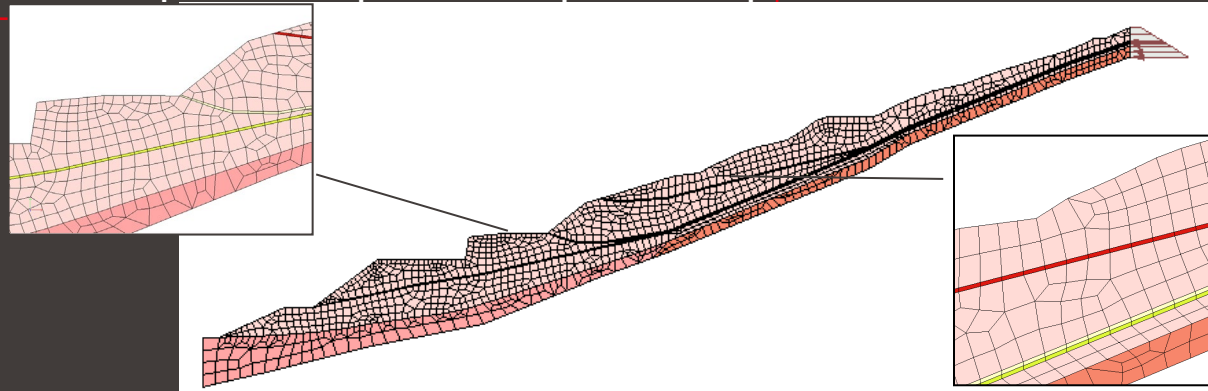
- Unstructured mesh with 1554 isoparametric Q4 elements (1694 nodes)
- Generalized effective stress concept (Nuth and Laloui, 2008)

$$\sigma'_{ij} = \sigma_{net\ ij} + S_r s \delta_{ij}$$

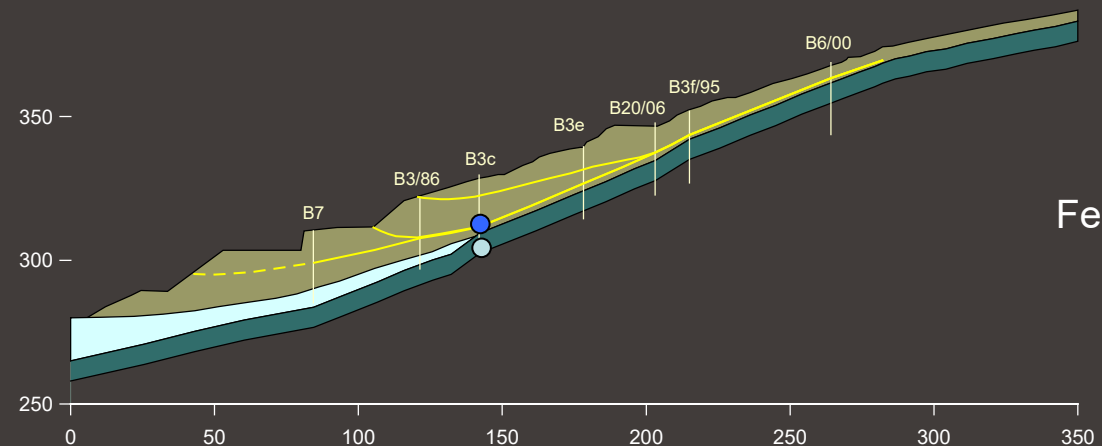
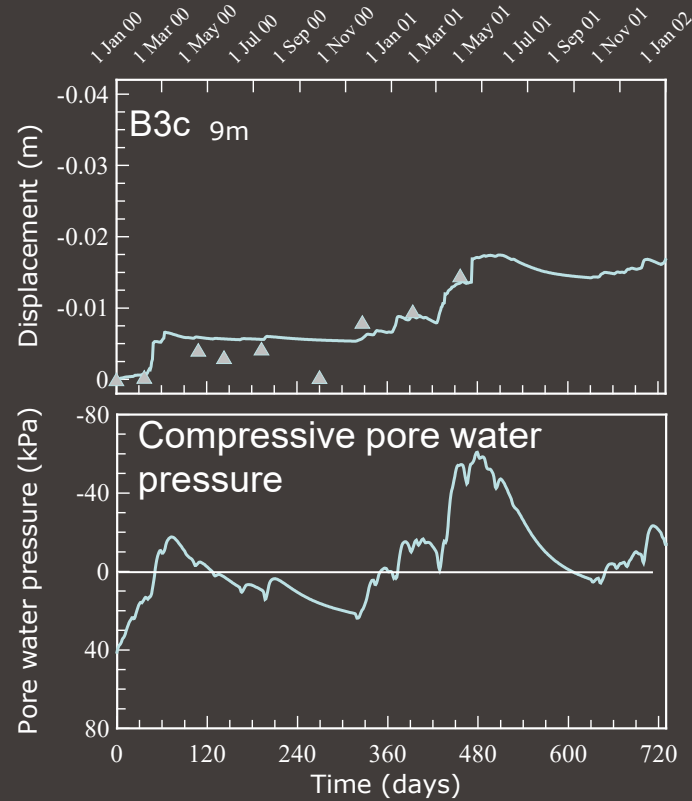
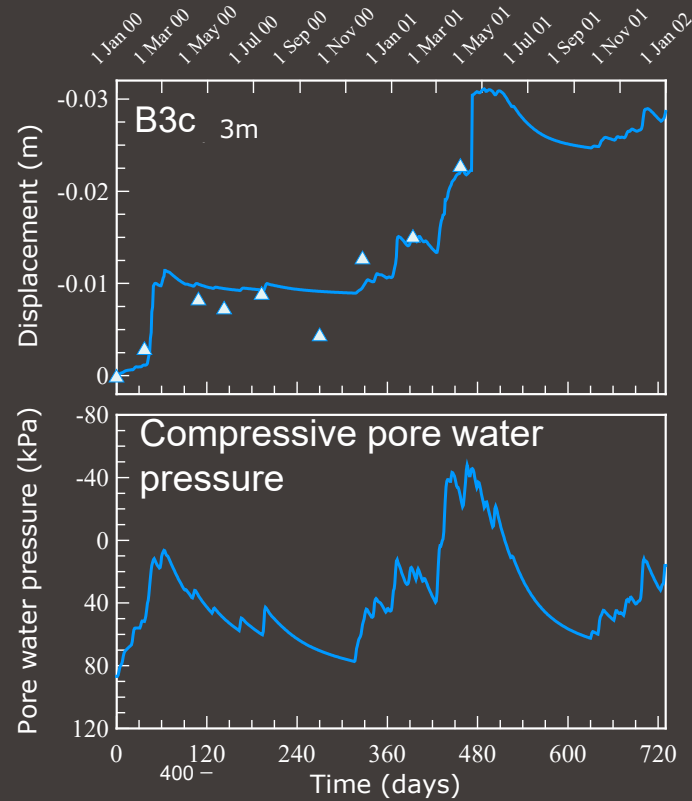
- Pore water pressure imposition at nodes at each computation instant
- Seepage elements to prevent indefinite pore water pressure increasing by pounding

Constitutive parameters assumed at the end of the calibration process (93 different combinations have been tested)

MATERIAL	Landslide body	Alluvium	Bedrock	Sliding surface 1	Sliding surface 2	Sliding surface 3
Constitutive law	E	E	E	CC	CC	CC
E' (kPa)	60'000	200'000	500'000	-	-	-
ν'	0.3	0.3	0.3	-	-	-
lambda	-	-	-	0.1	0.1	0.1
kappa	-	-	-	0.05	0.05	0.05
c'	-	-	-	-	-	-
ϕ' (M)	-	-	-	23° (0.9)	24° (0.95)	25.5° (1)
ψ	-	-	-	Assoc	Assoc	Assoc
γ (kN/m ³)	17	17	20	17	17	17
e ₀	0.65	0.6	0.5	0.65	0.65	0.65
K ₀	0.8	0.8	0.8	0.8	0.8	0.8
S _{res}	0	0	0	0	0	0
α (1/m)	1	1	1	1	1	1



Model calibration: Simulations of displacements for the years 2000 - 2001



Ferrari et al. / CMES (2009)

Model calibration and validation

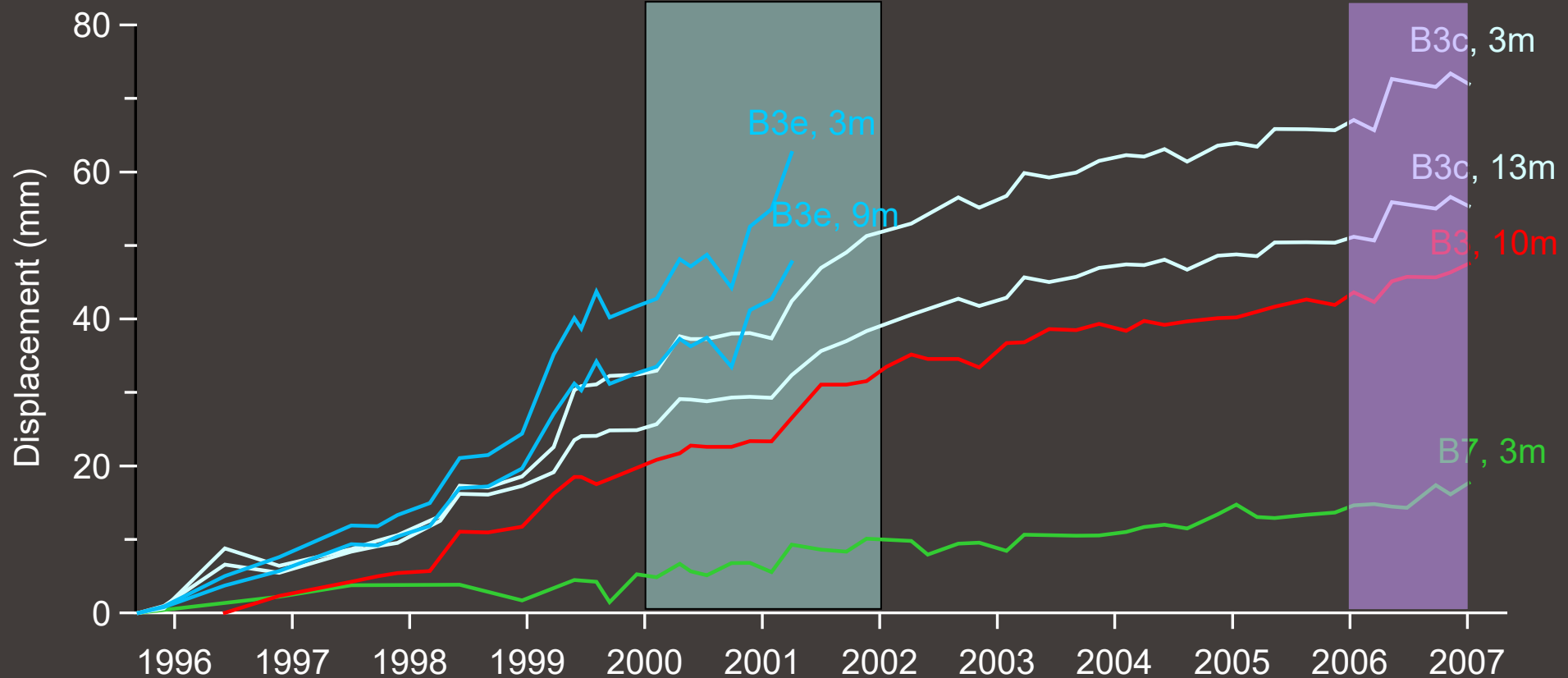
Hydrological model calibration for the period 2000 - 2001

Pore water pressure distribution are obtained as an output of the calibration

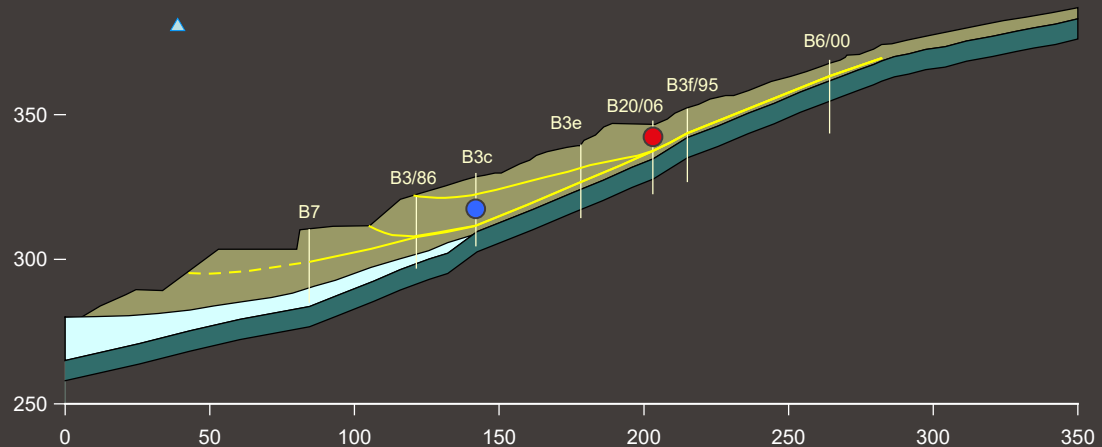
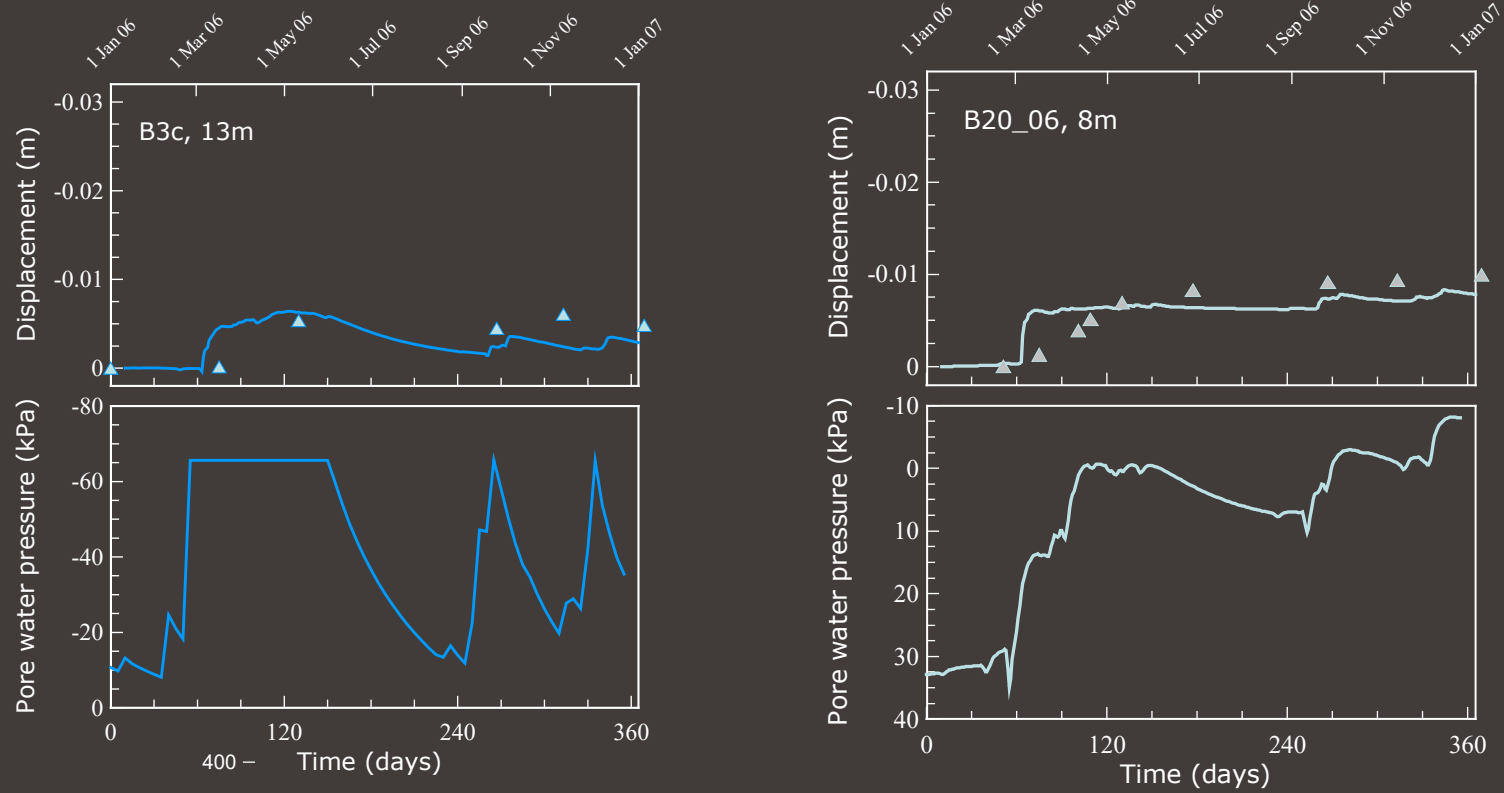
Pore water pressure are predicted for the year 2006

Geomechanical model calibration for the period 2000-2001

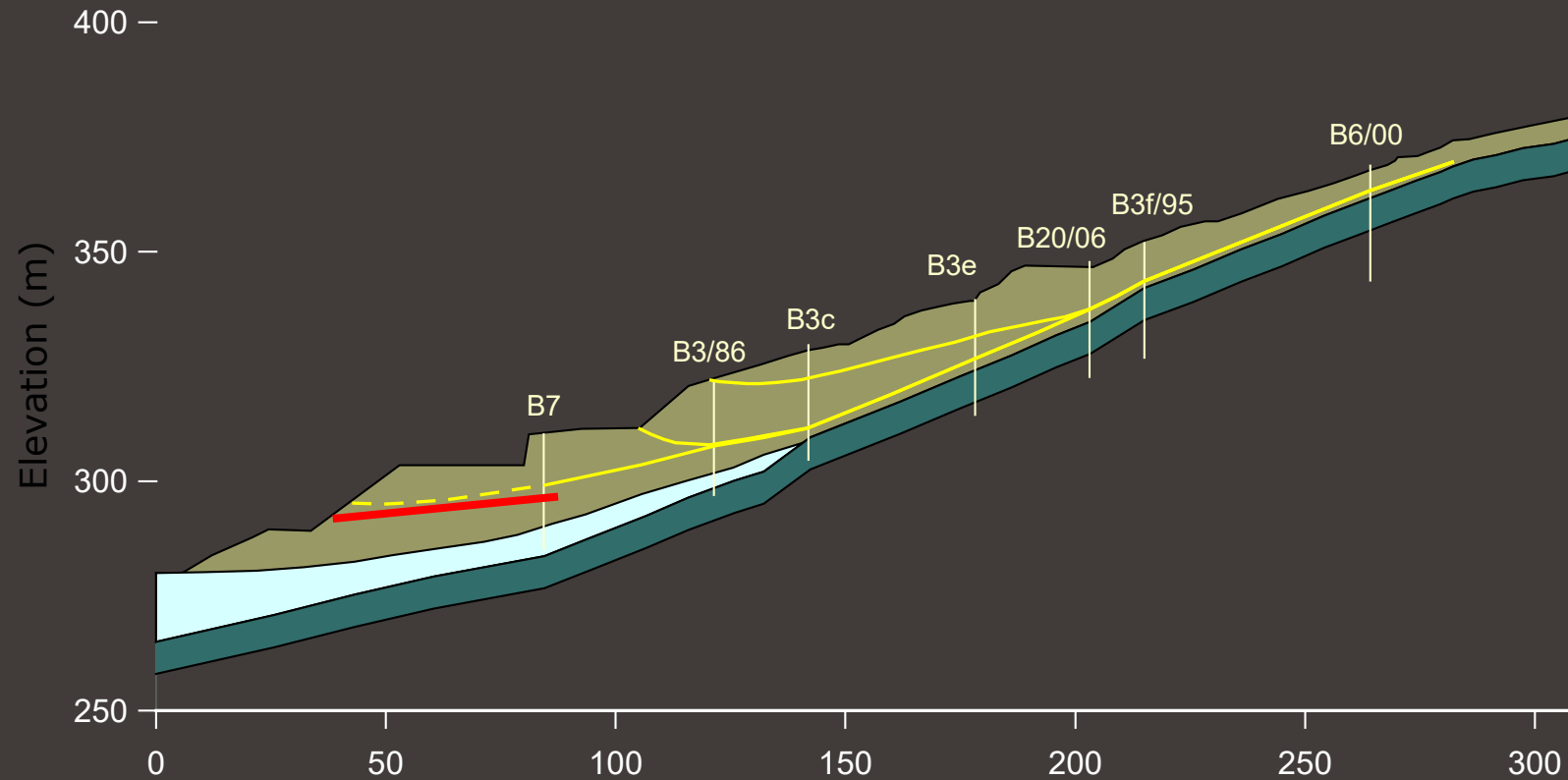
Geomechanical model validation for the year 2006



Model validation: Simulations of displacements for the year 2006

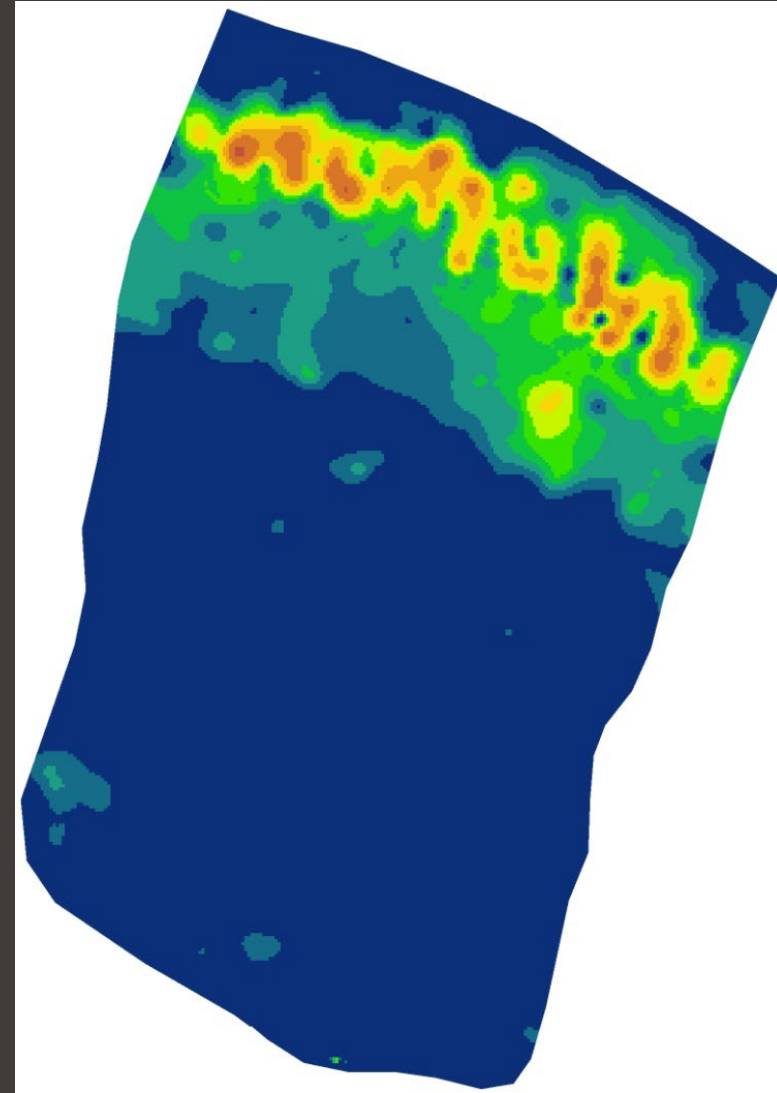
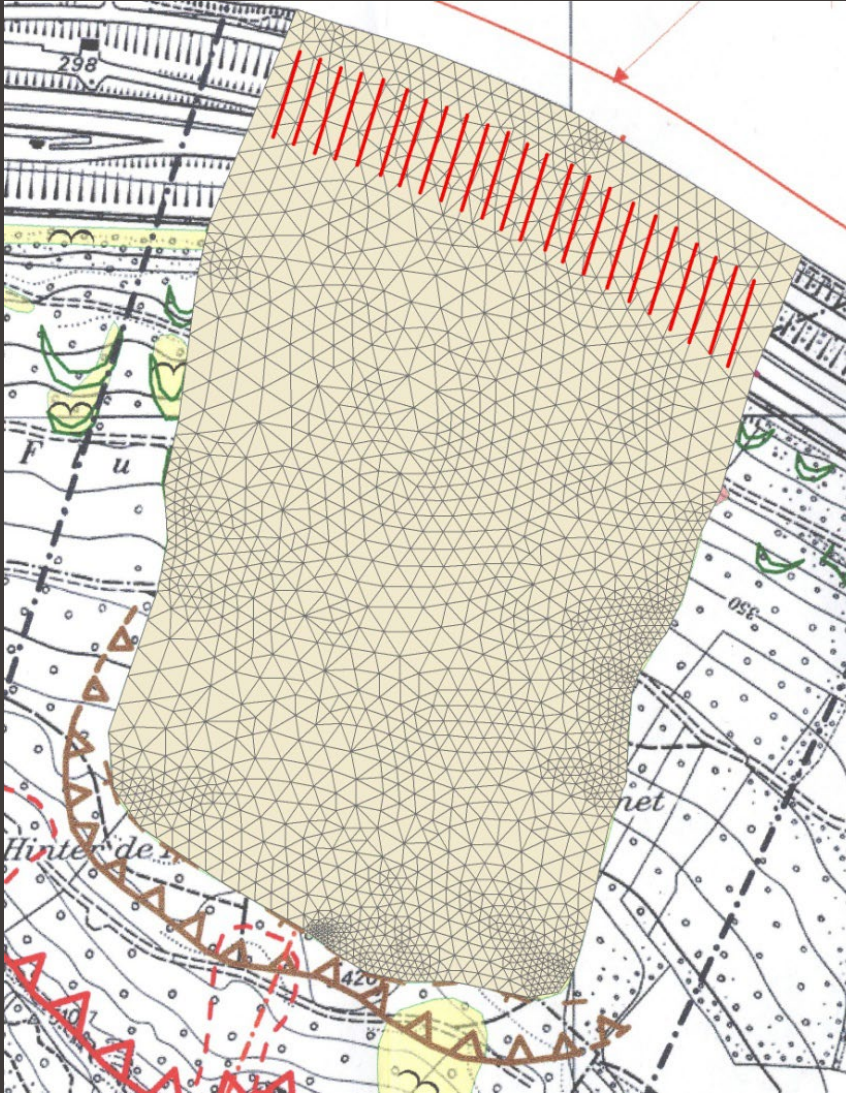


Possible solutions: sub-horizontal drainage system



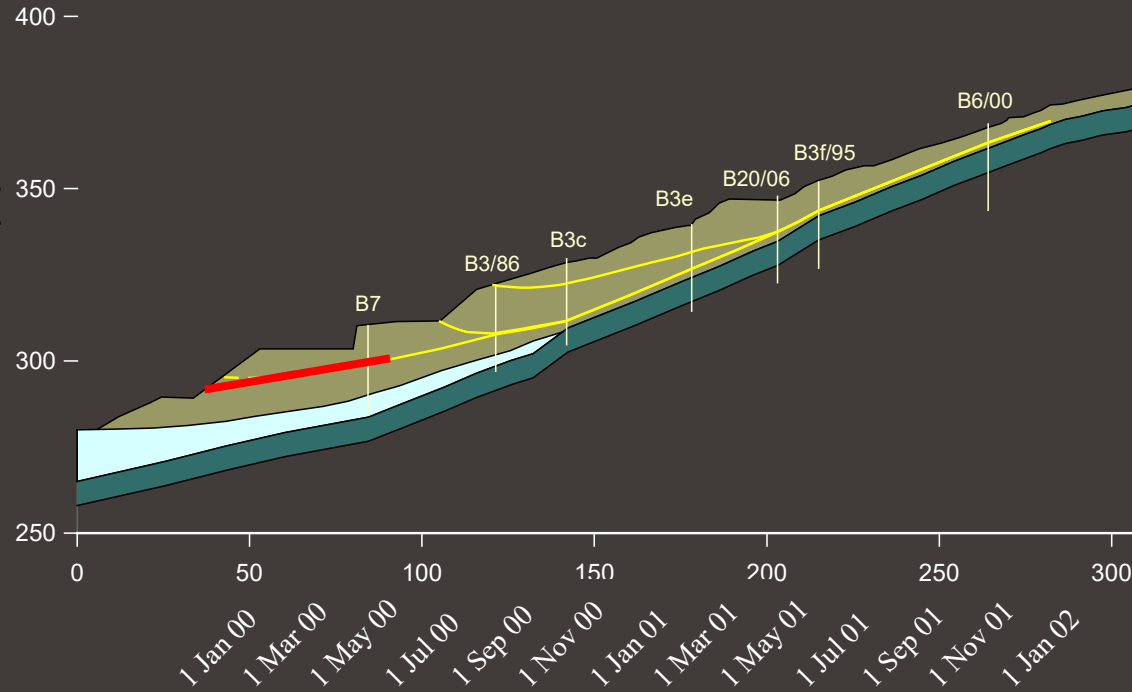
Decrease the pore water pressures at the slope toe below the deep seated sliding surface by means of a drain system: length 50m, inclination 3°

Possible solutions: sub-horizontal drainage system

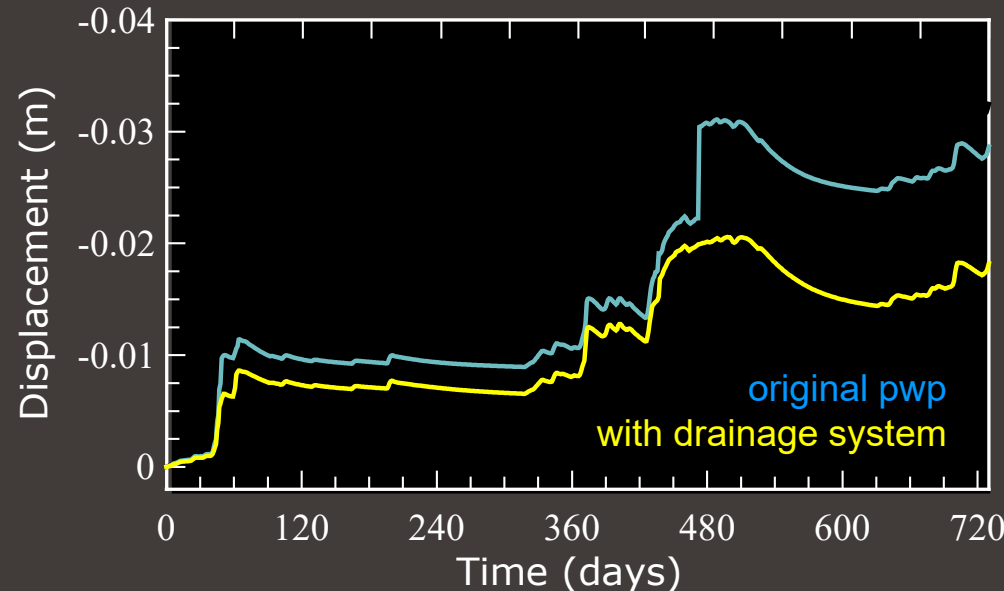


Drainage system in the hydro-geological model: reduction of the total head for the period 2000-2001 (0, blue – 16 m, red) (Geolep/EPFL)

Possible solutions: sub-horizontal drainage system



Decrease the pore water pressures at the slope toe below the deep seated sliding surface by means of a drain system: length 50m, inclination 3°



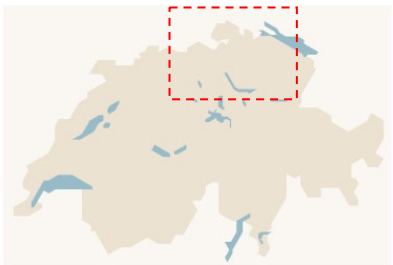
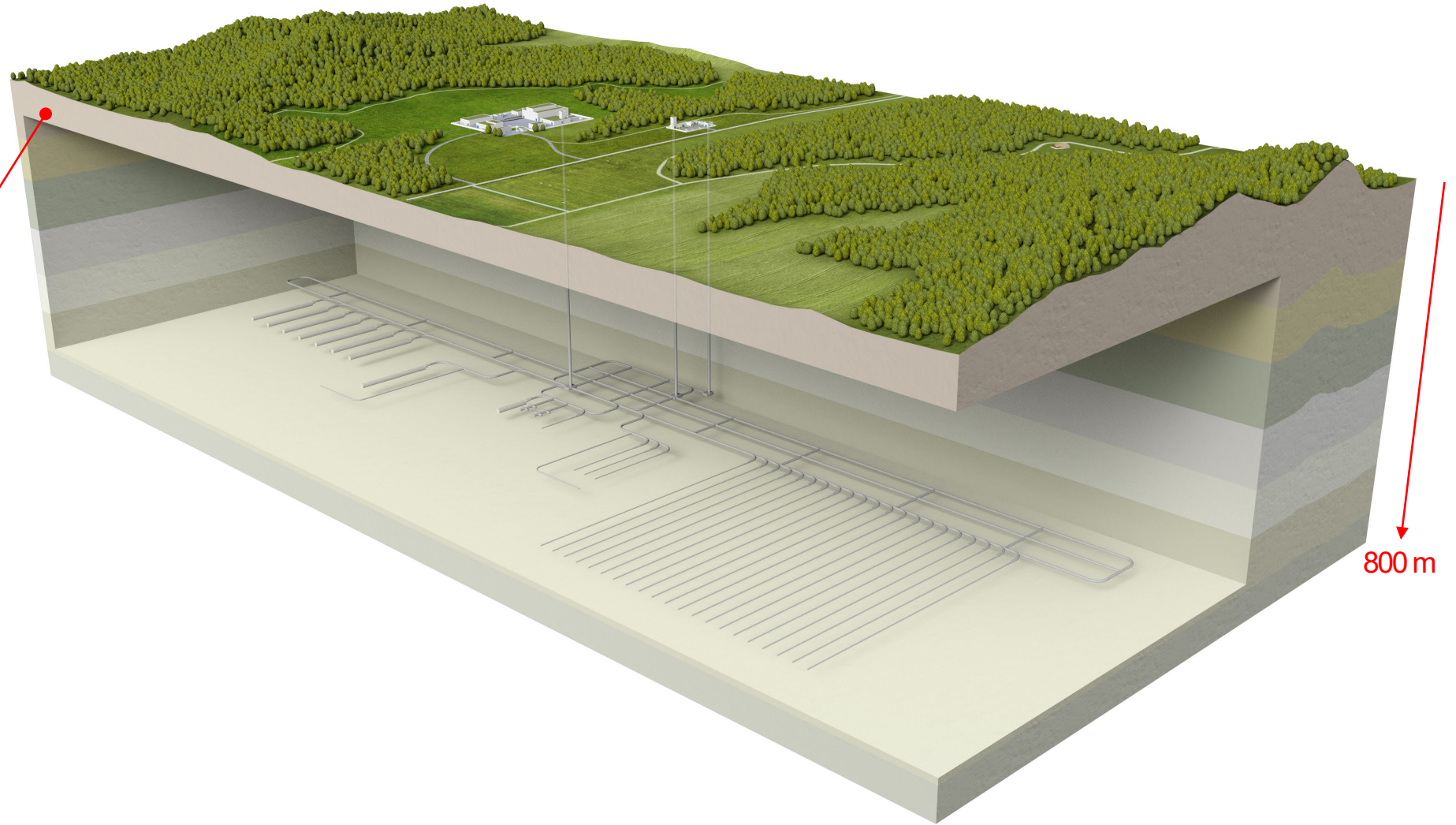
Displacement trend in B3e,3m the with the drainage system

Example 3

THM FORMULATION : NUCLEAR WASTE REPOSITORY

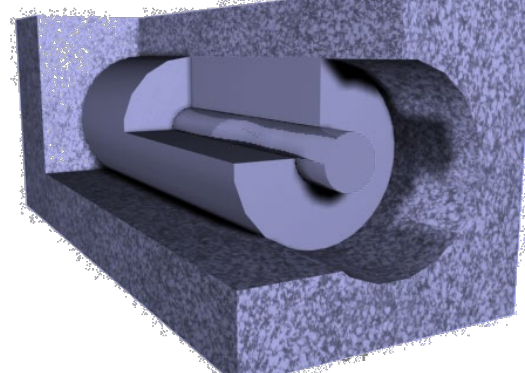
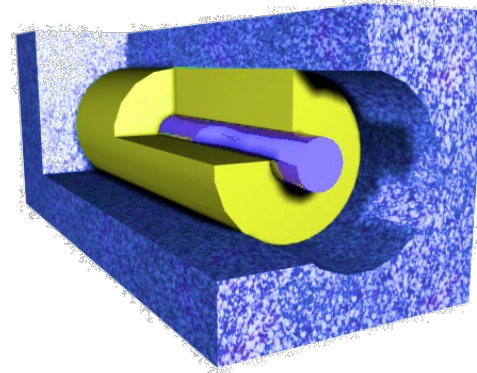
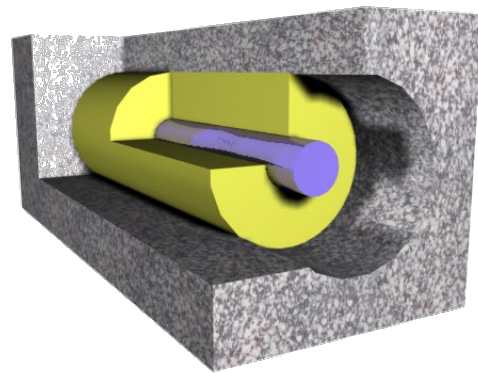
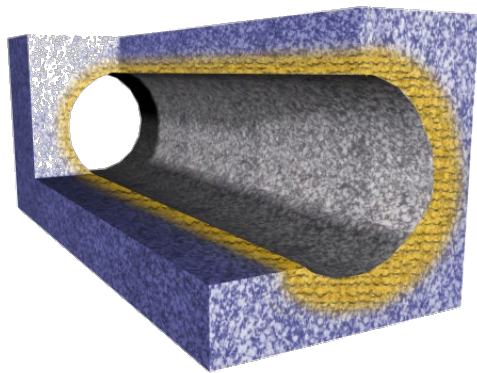
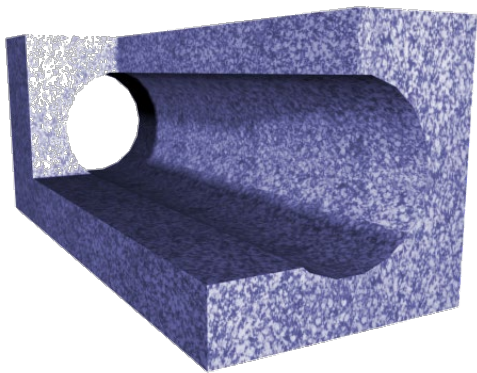
Deep repository systems

The future Swiss repository



Geomechanics for repository systems

Multiphysical processes across time



Deep repository systems

Opalinus Clay Shale

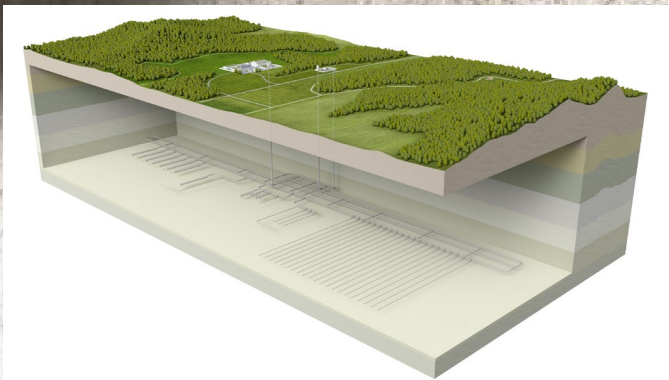
Granual bentonite

Spent fuel

0.5 m

Metallic canister

Bentonite in blocks



Governing equations

Solid:
$$\frac{\partial}{\partial t} (\rho_s(1-n)V) = 0$$

Water:
$$\frac{\partial}{\partial t} (\rho_w n S_r) + \frac{\partial}{\partial x_i} (\rho_w f_{l,i}) - Q_w +$$

$$\frac{\partial}{\partial t} (\rho_v n (1 - S_r)) + \frac{\partial}{\partial x_i} (i_{v,i} + \rho_v f_{g,i}) - Q_v = 0$$

Vapor storage *Vapor flux*

Air (dry):
$$\frac{\partial}{\partial t} (\rho_a n (1 - S_r)) + \frac{\partial}{\partial x_i} (i_{a,i} + \rho_a f_{g,i}) - Q_a +$$

$$\frac{\partial}{\partial t} (H \rho_a n S_r) + \frac{\partial}{\partial x_i} (H \rho_a f_{l,i}) - Q_{da} = 0$$

Dissolved water *Dissolved water advection*

$$\frac{\partial S_T}{\partial t} + L \frac{\partial}{\partial t} (\rho_v n (1 - S_r)) + \frac{\partial f_{T,i}}{\partial x_i} +$$

Latent heat *Conduction*

$$L \frac{\partial}{\partial x_i} (i_{v,i} + \rho_v f_{g,i}) - Q_T = 0$$

Advection

$$\frac{\partial \sigma_{ij}}{\partial x_i} + b_j = 0$$

Variables

Displacements: \mathbf{u}
 Liquid pressure: p_w
 Gas pressure: p_a
 Temperature: T

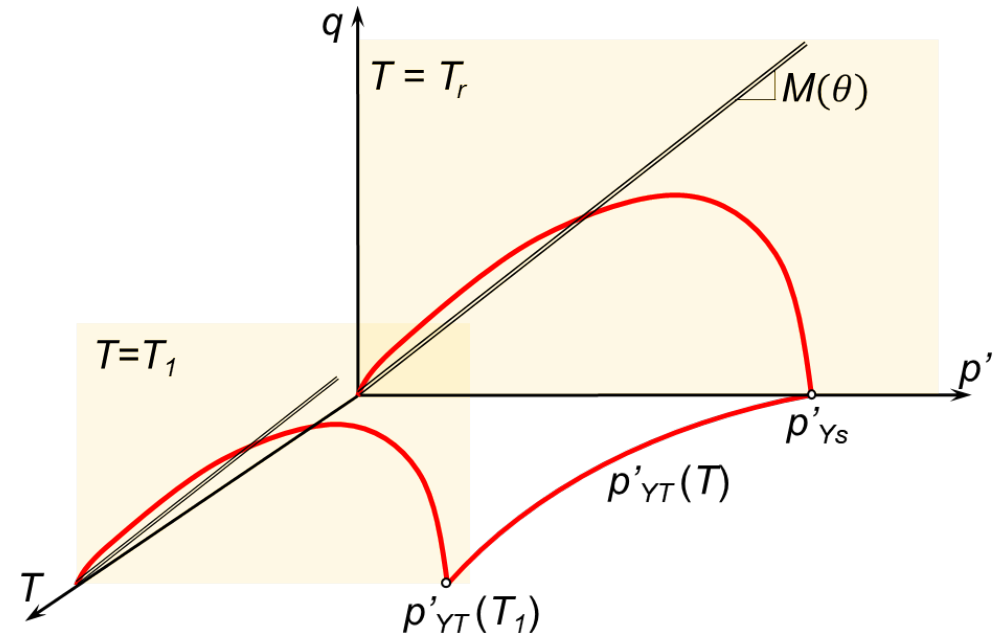
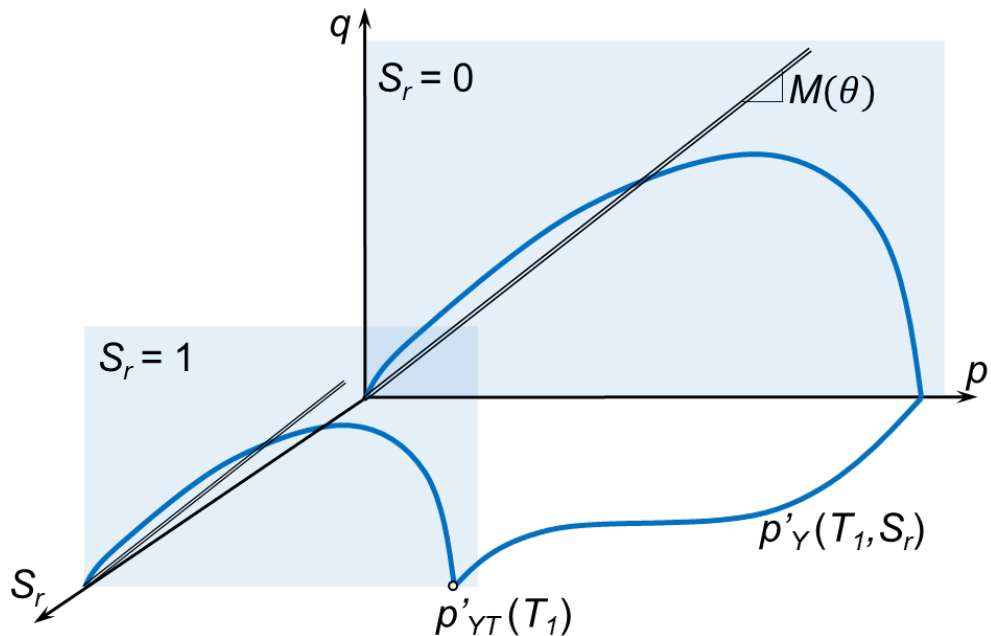
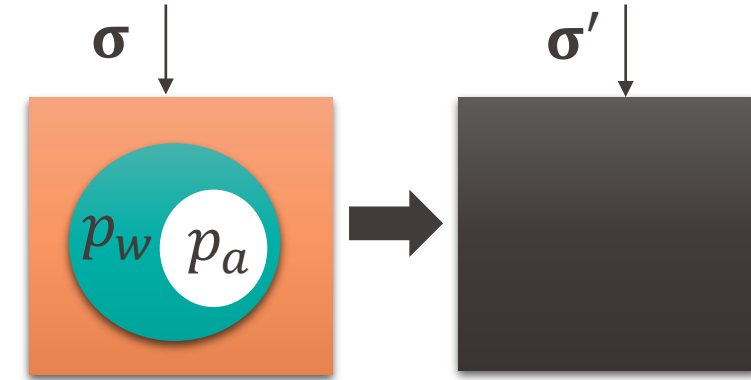
Multiphysical constitutive modelling of bentonite

- Two constitutive variables are used to model bentonite behaviour under variably saturated states:

$$1) \quad \sigma' = \sigma - [p_a - S_r(p_a - p_w)]\mathbf{I}$$

$$2) \quad S_r = e_w/e$$

- Yield surfaces extended to variable hydration states and non-isothermal conditions



Multiphysical constitutive modelling of bentonite

- Thermo-elasticity:

$$dp' = \frac{p'}{\kappa} d\epsilon_v^e - [\beta_{T0} - \beta_{T1}(T - T_r)]dT, \quad dq = \frac{p'}{\kappa} \frac{9(1 - \nu)}{2(1 + \nu)} d\epsilon_q^e$$

- Yield function in the (σ', S_r, T) space:

$$f = q^2 - M^2 \left[\alpha + (1 - \alpha) \left(\frac{2p'}{p'_Y} \right) \right]^2 p'(p'_Y - p')$$

$$p'_Y = p'_r \left(\frac{p'_{YT}}{p'_r} \right)^{\frac{\lambda_s - \kappa}{\lambda - \kappa}}$$

$$M(\theta) = a_L [1 + b_L \sin(3\theta)]^{n_L}$$

$$\lambda = \lambda_s - r(\lambda_s - \kappa)(1 - S_r \zeta)^\xi \quad p'_{YT} = p'_{Ys} \left[1 - \gamma_T \ln \left(\frac{T}{T_r} \right) \right]$$

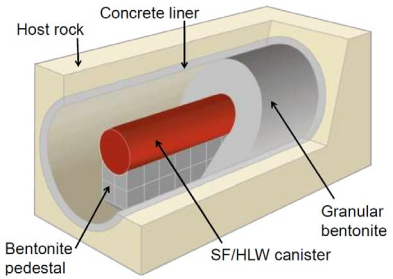
- Flow rule and hardening law:

$$d\epsilon_v^p = -d\Lambda(p' - p'_Y/2), \quad d\epsilon_d^p = -d\Lambda \frac{q}{M^2 \Pi^2}, \quad \frac{dp'_{Ys}}{p'_{Ys}} = \frac{d\epsilon_v^p}{\lambda_s - \kappa}$$

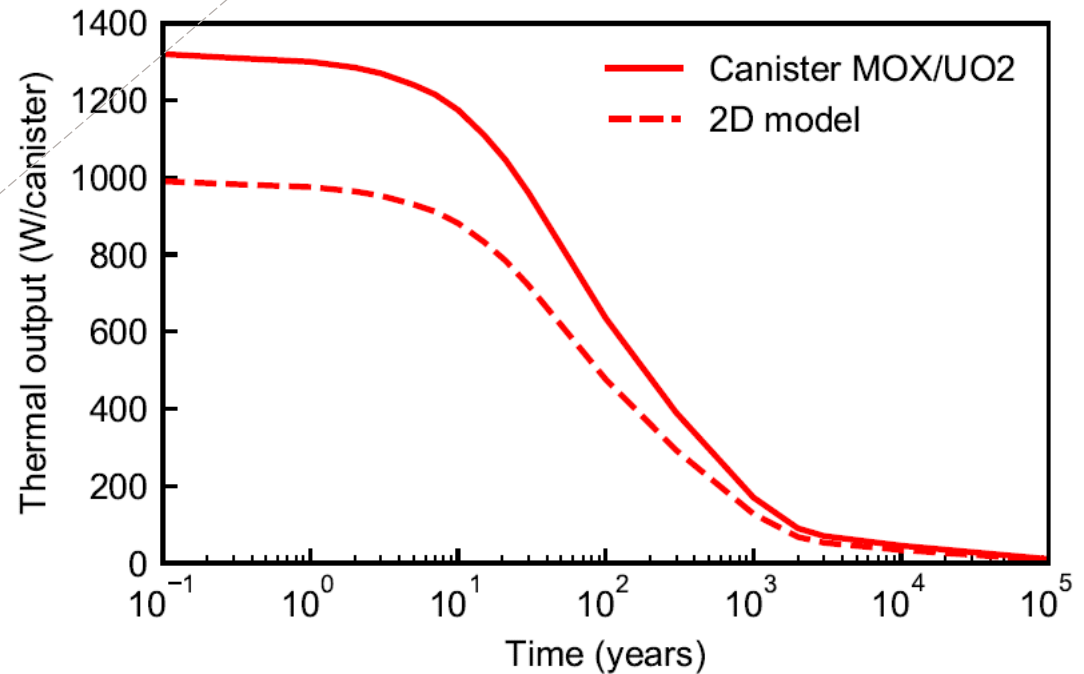
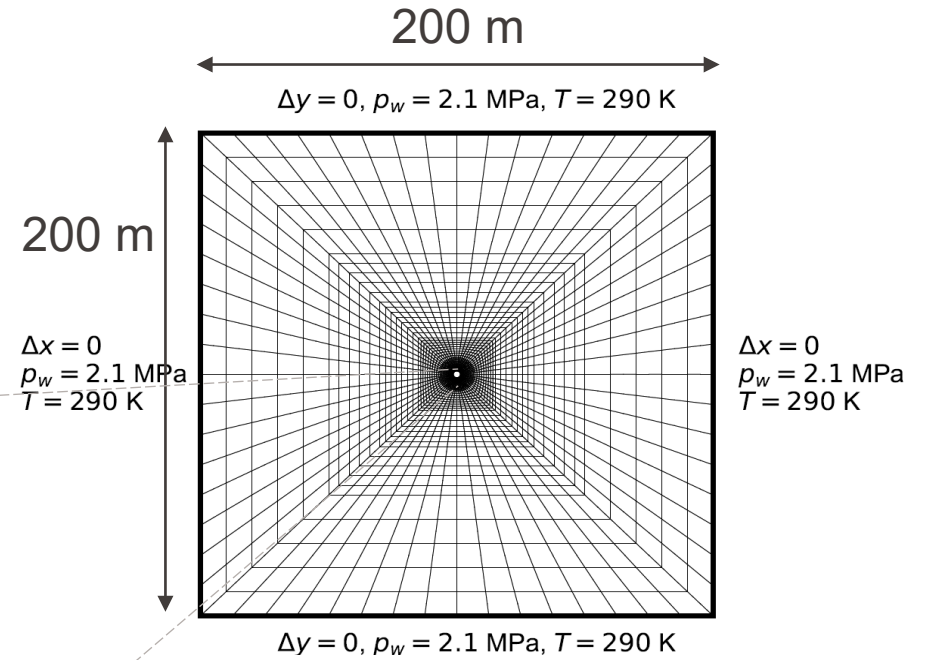
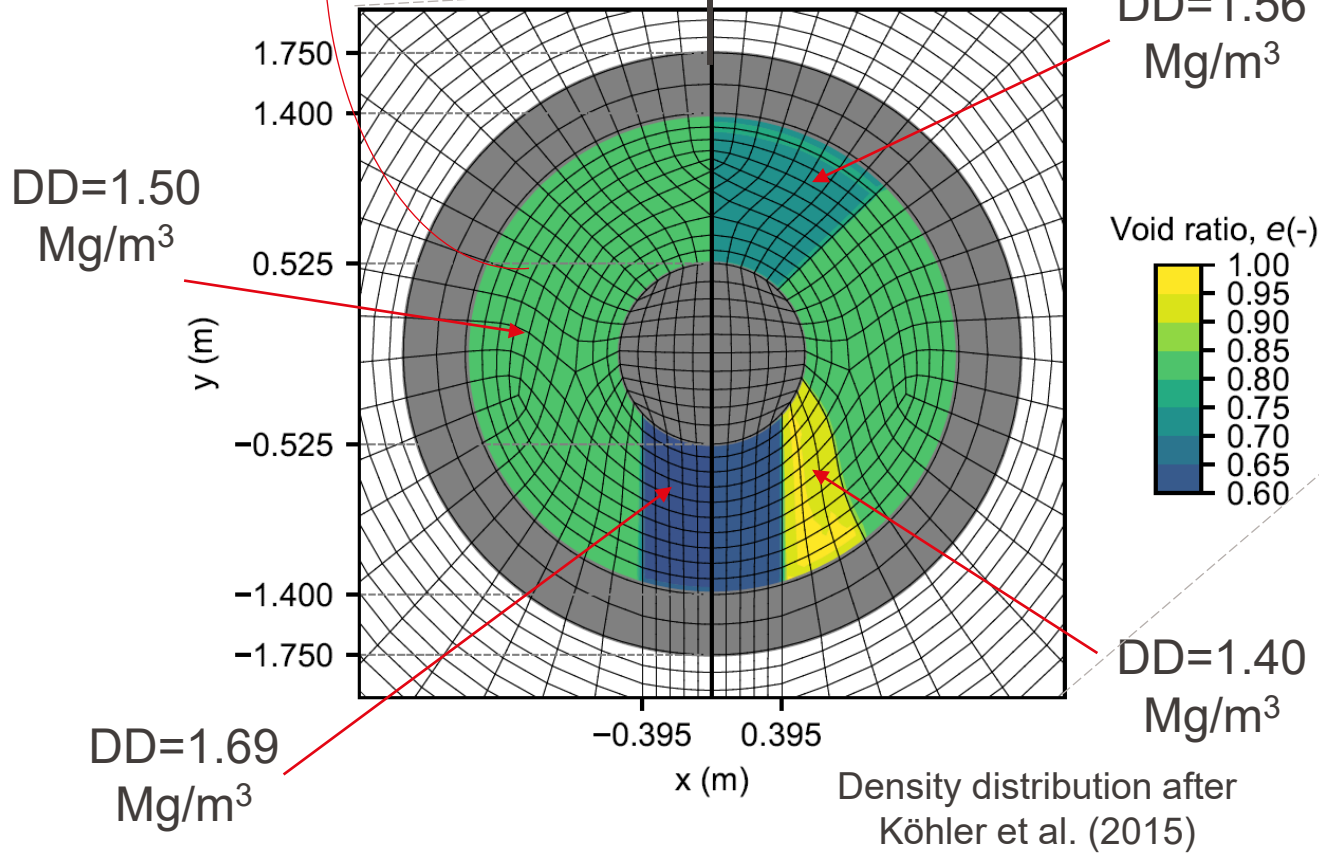
- (Bosch, Ferrari and Laloui, C&G, 2021)

Elasticity	
Elastic compressibility	κ
Poisson's ratio	ν
Thermo - elasticity	
Thermal expansion coefficient	β_{T0}
Evolution of thermal expansion coefficient	β_{T1}
Plasticity	
Saturated elastoplastic compressibility	λ_s
Shear strength angle for TXC	$a_L(\phi_c), b_L(\phi_c)$
Shear strength angle for TXE	$a_L(\phi_e), b_L(\phi_e)$
Van Eekelen exponent	n_L
Non-associativity parameter	α
Unsaturated plasticity	
Reference pressure for unsaturated states	p'_r
Compressibility ratio between $S_r=0$ and $S_r=1$	r
Evolution of compressibility with S_r (1)	ζ
Evolution of compressibility with S_r (2)	ξ
Thermo-plasticity	
Thermal yield parameter	γ_T

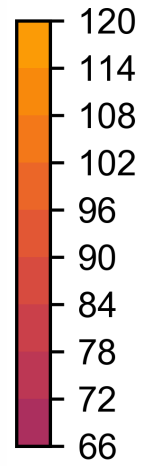
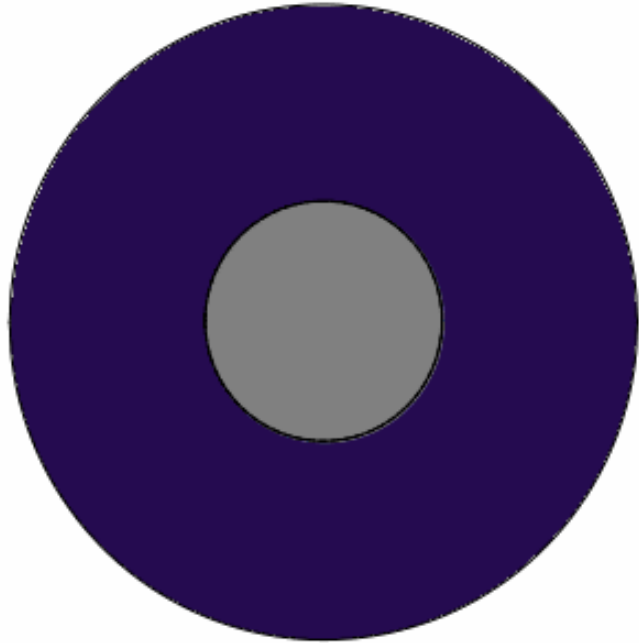
Model of NAGRA cell



Base case Segregation case

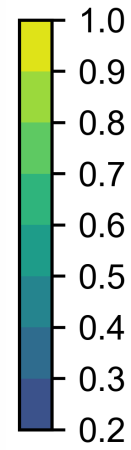
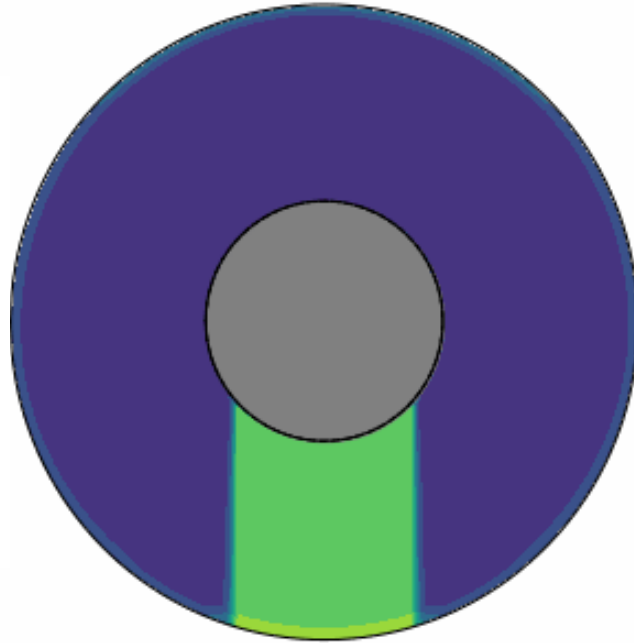


Temperature (°C)

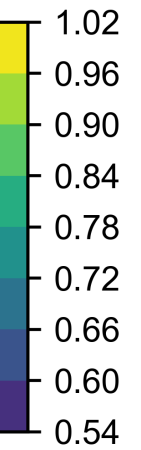
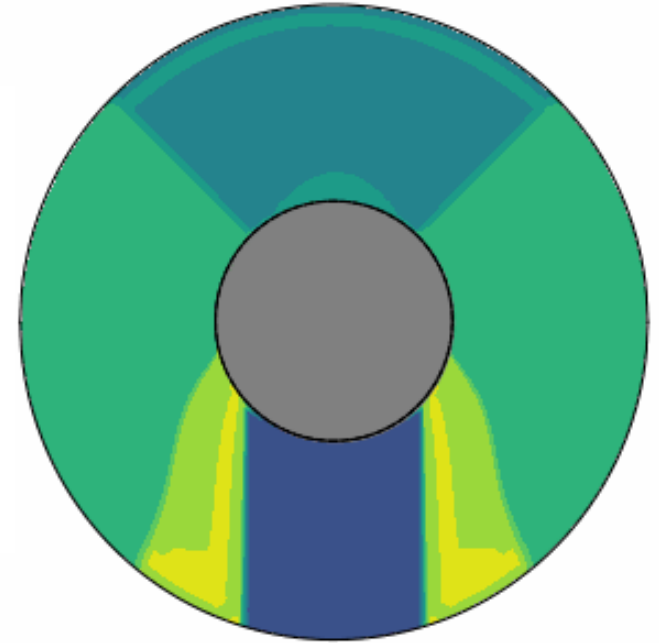


Year 0.0

Sr (-)

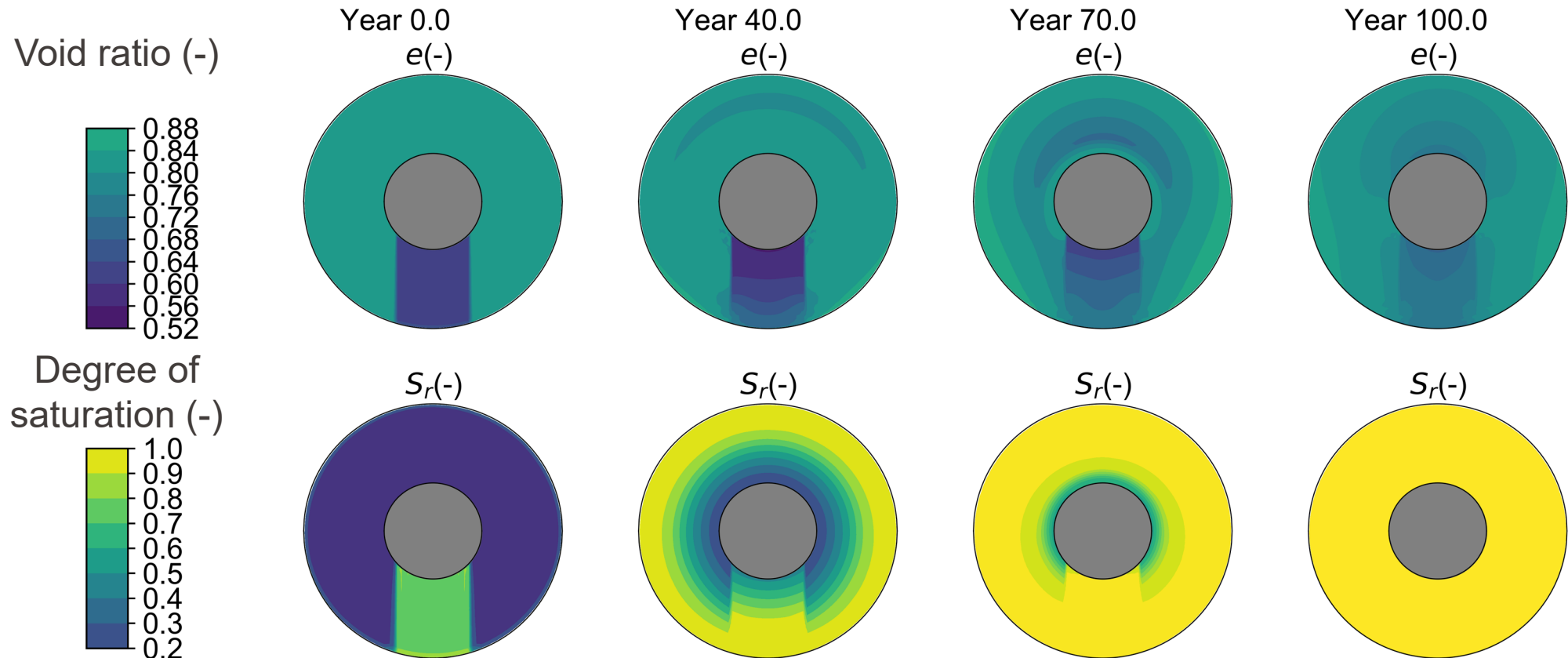


Void ratio (-)



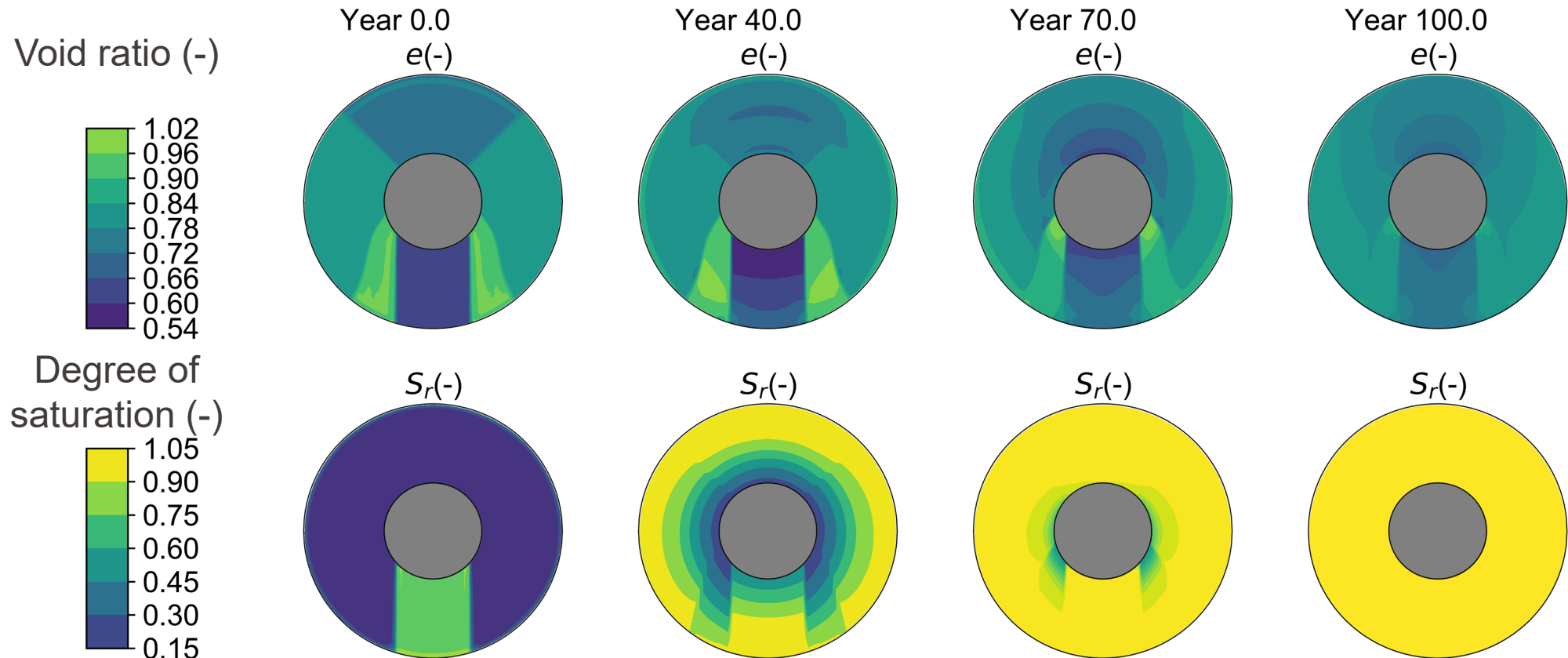
Homogenisation process

Case 1: No initial segregation of granular bentonite



Homogenisation process

Case 2: Initial segregation of granular bentonite



Thank you for your attention

